Verification under Weak Consistency and Recent Decidability Results in Verification

Roland Meyer, TU Braunschweig

Joint work with the best PhD students in the world :)

IFIP WG 2.3, Athens, May 2025

Verification under Weak Consistency

Introductory example

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         x = y = 0
(a) x = 1;   |  (c) y = 1;
(b) r1 = y;  |  (d) r2 = x;
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…practice says otherwise!

Store buffering
x86/TSO

CPUs can buffer a store locally and only later flush it into main memory

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```
(I1) x = 0;          (I2) y = 0;
```

P0

```
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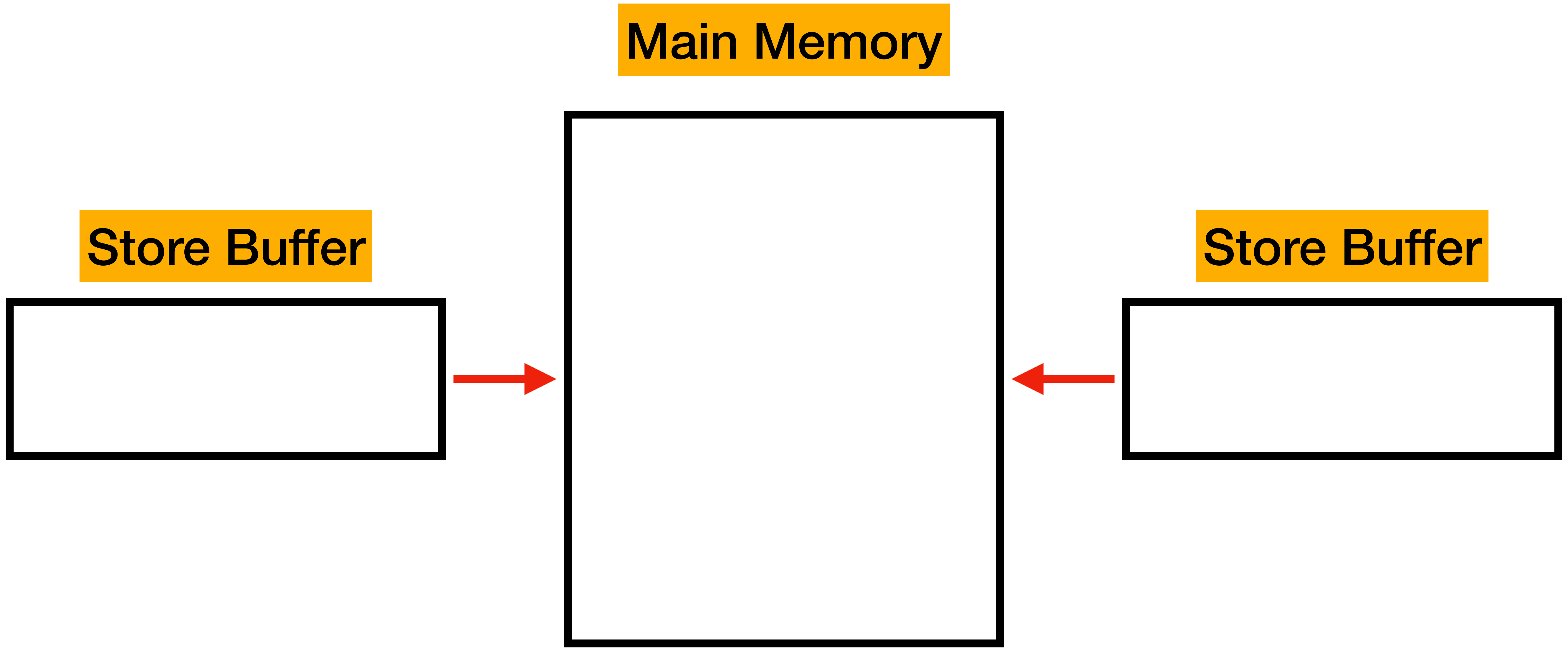
(b) r1 = y;
```



P1

```
(c) y = 1;

(d) r2 = x;
```

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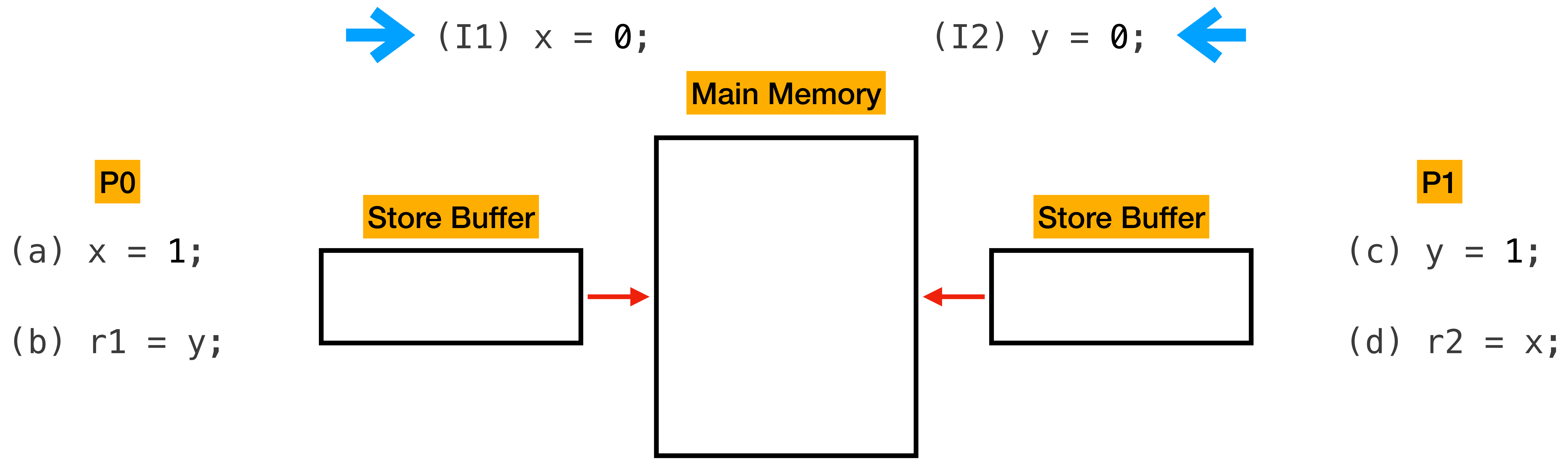

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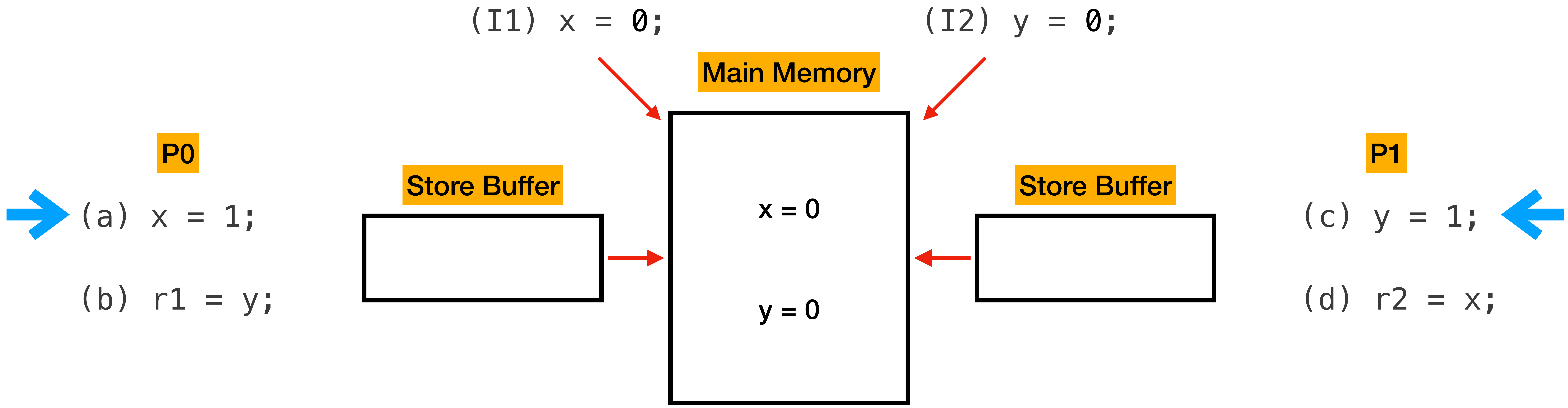

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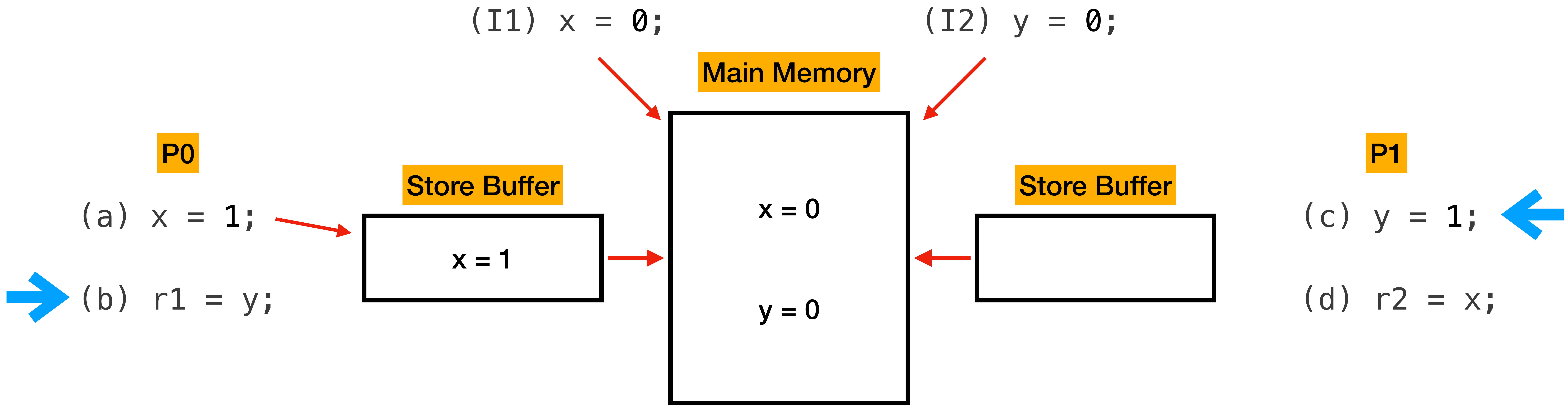

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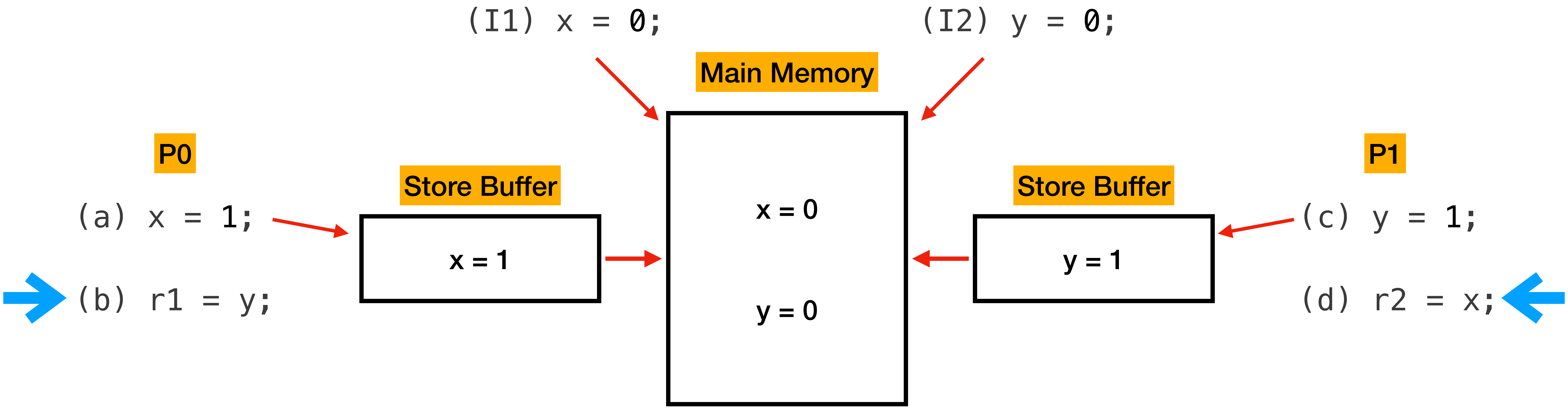

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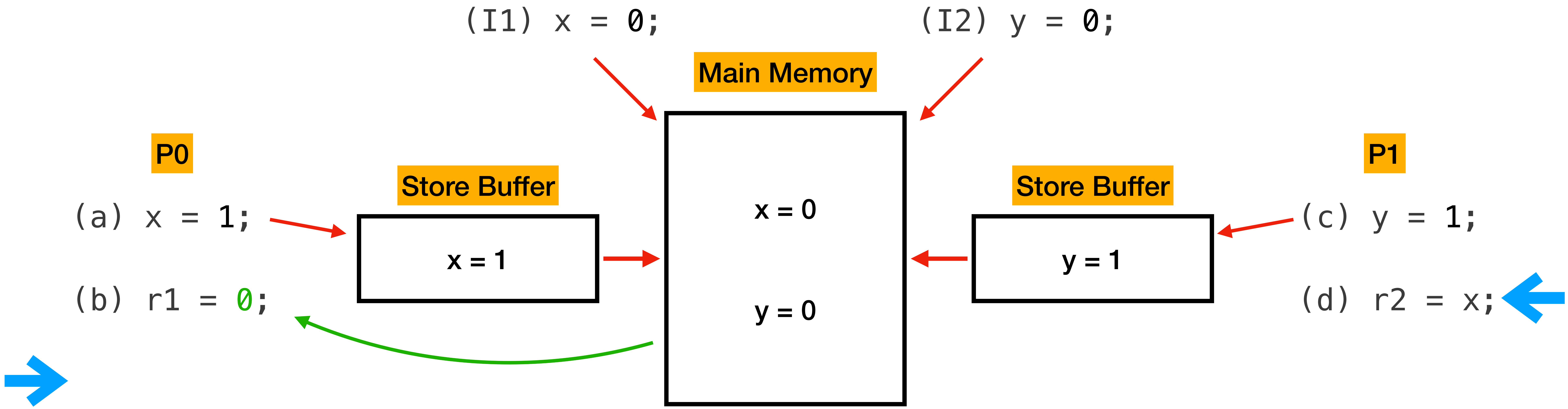

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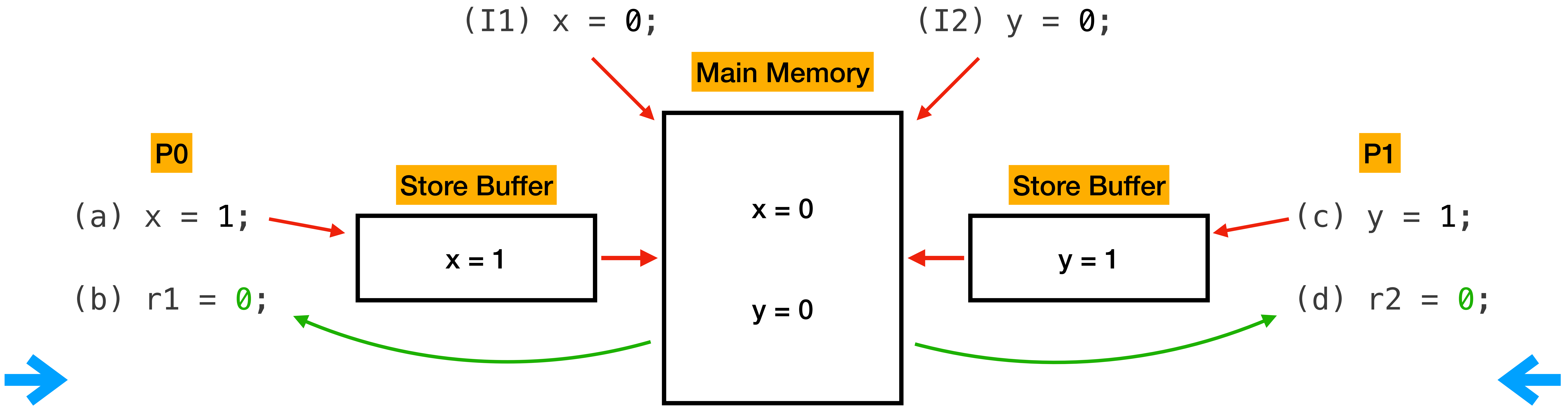

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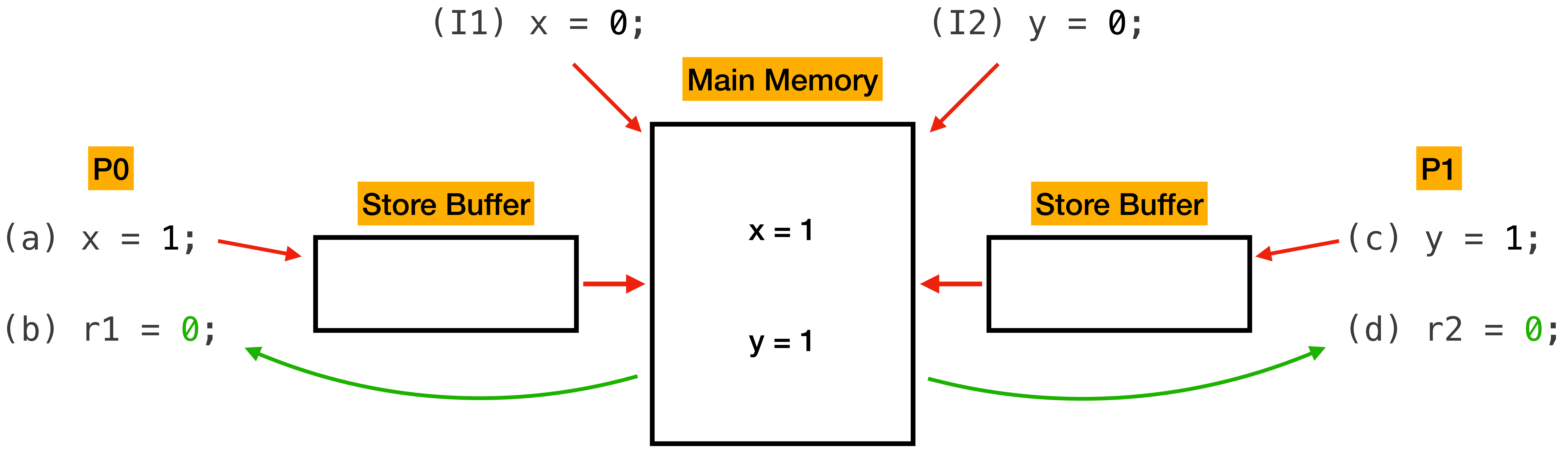

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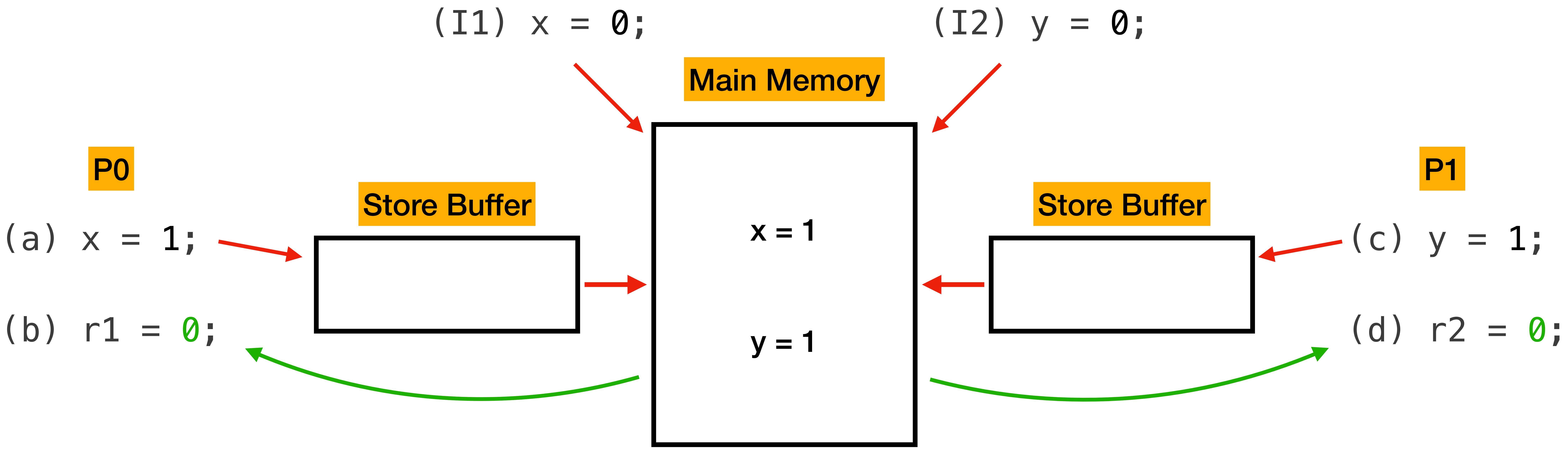

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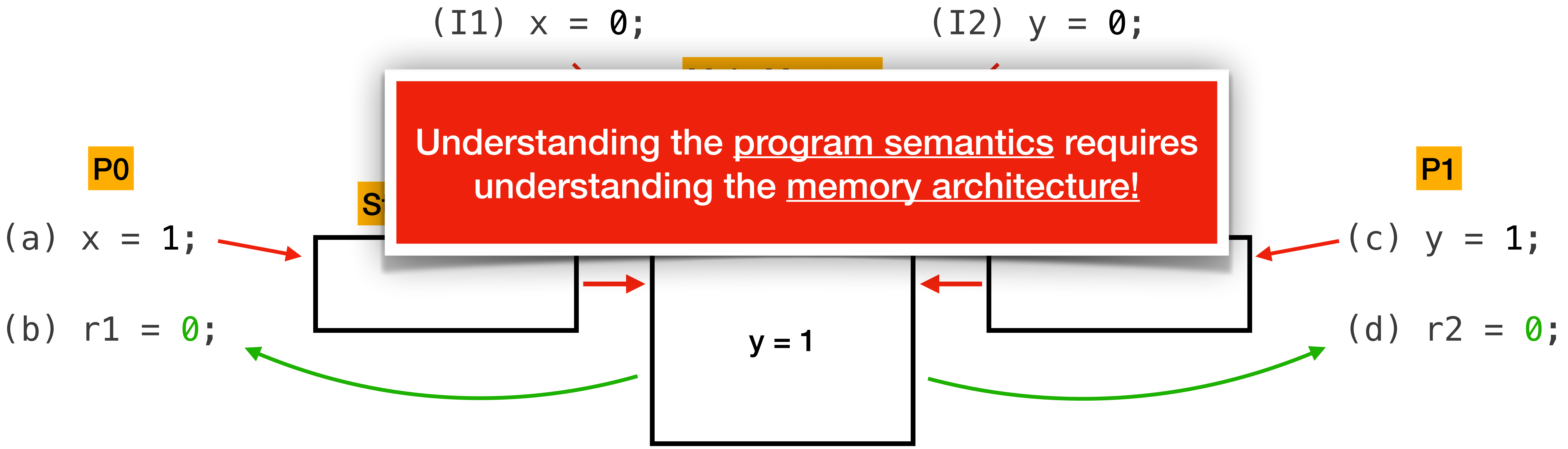

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Memory architectures (sketched)

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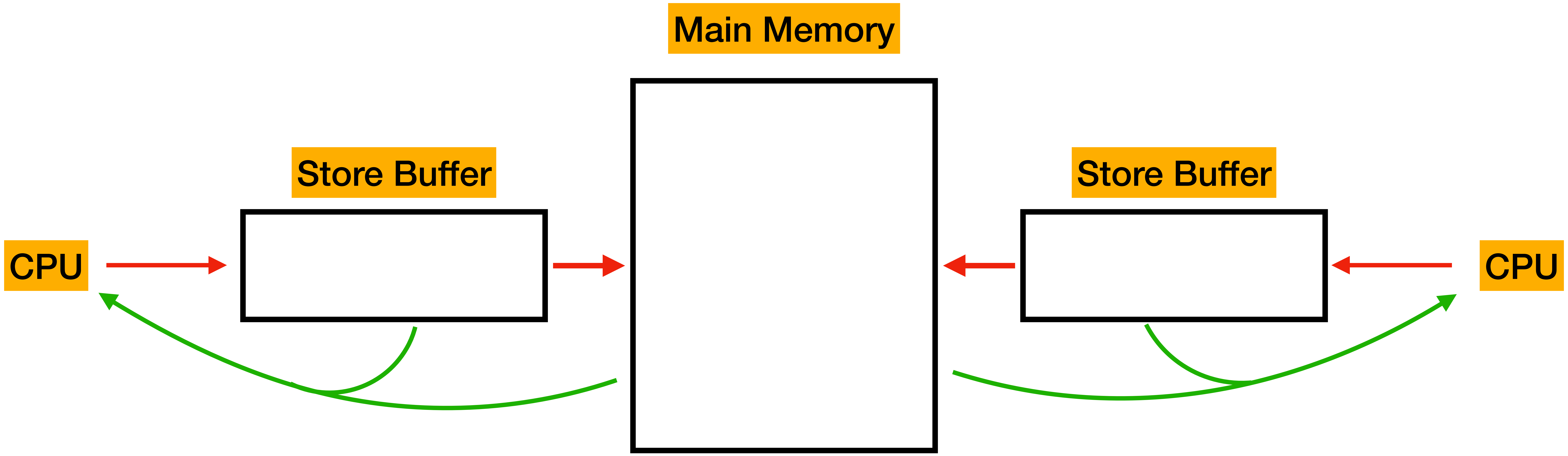

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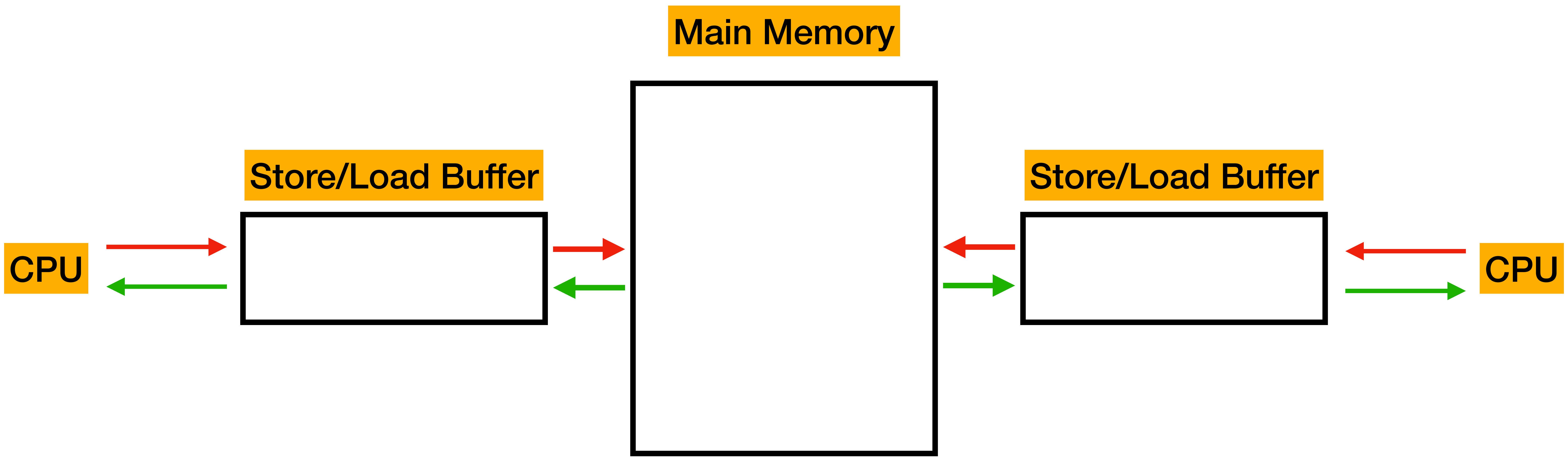
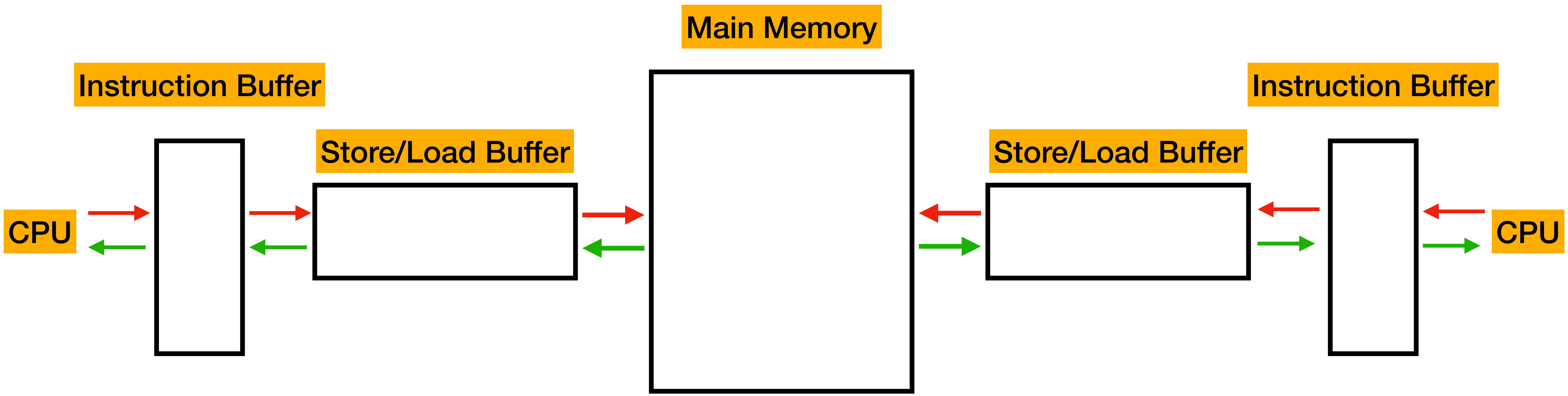


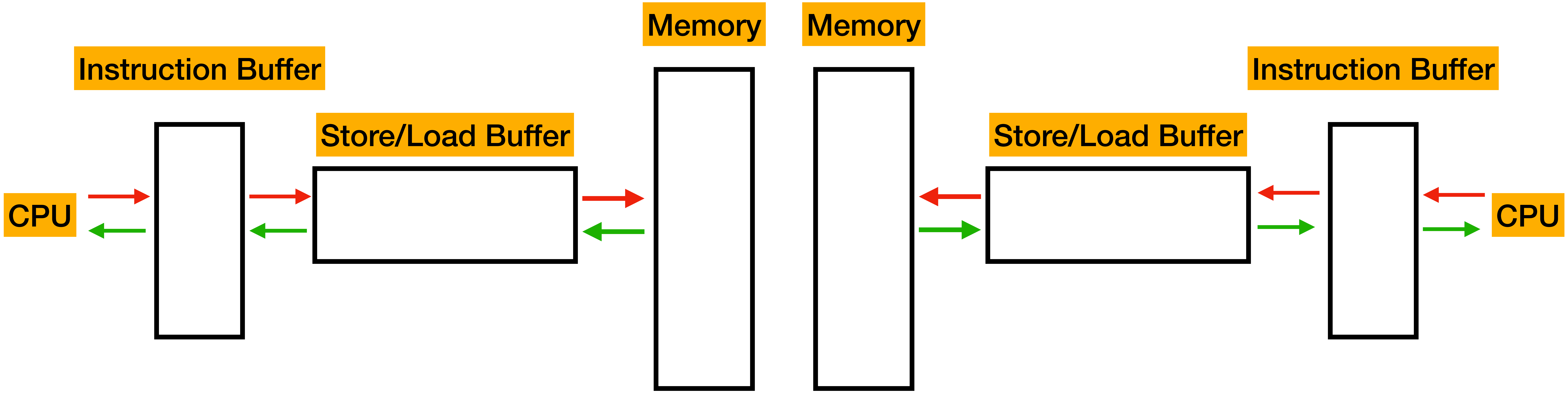
Memory architectures (sketched)

Store/Load buffer, instruction buffer



Memory architectures (sketched)

Store/Load buffer, instruction buffer, decentralised memory



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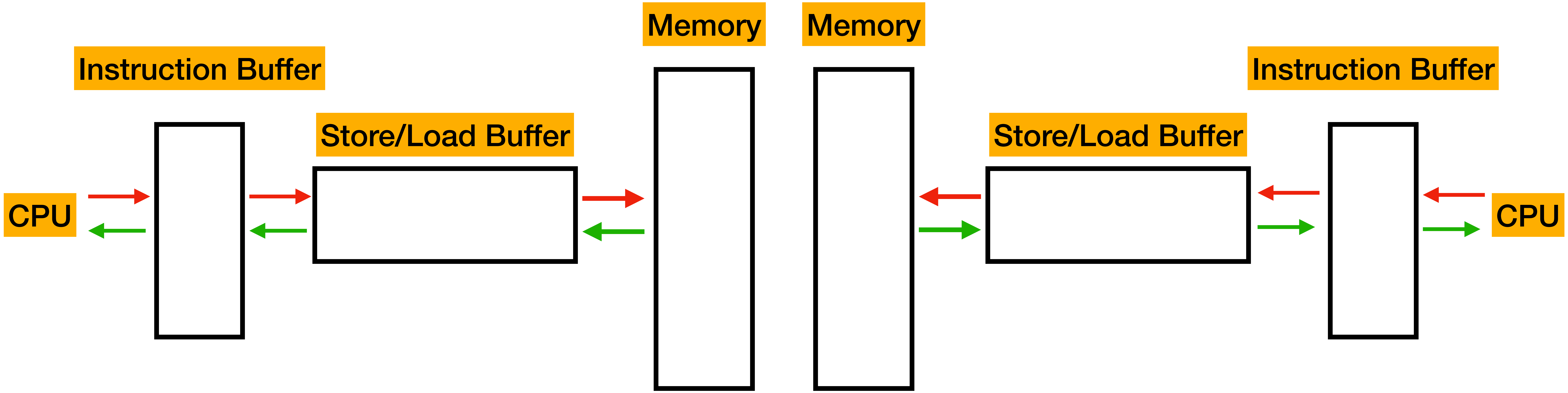

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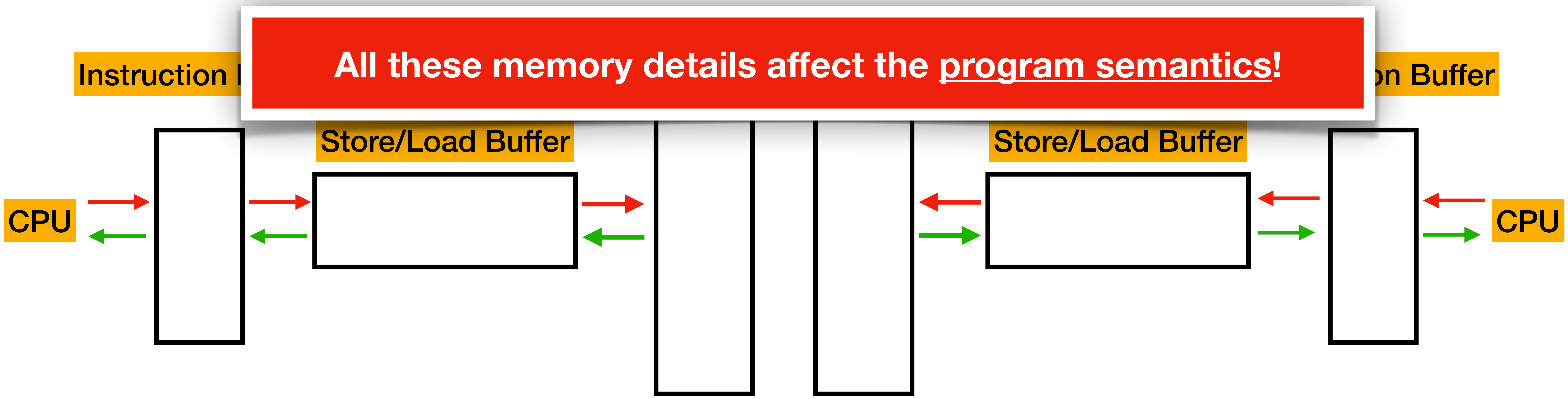

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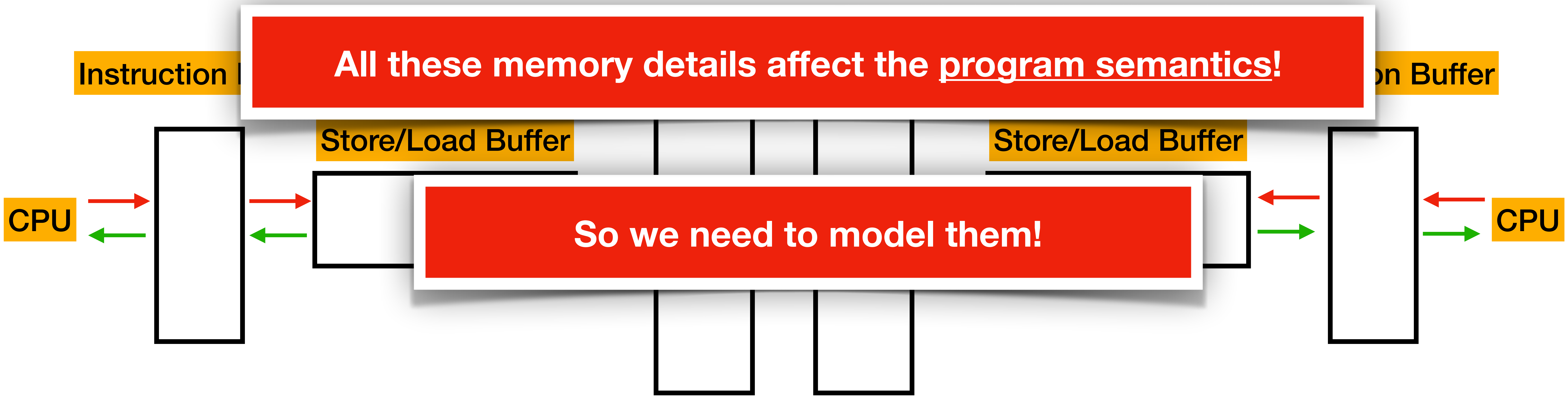

Program semantics

Capturing program semantics

Problems so far

Interleavings are insufficient to capture program behavior

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    - Forget about architectural details!
      Directly model observable behavior!

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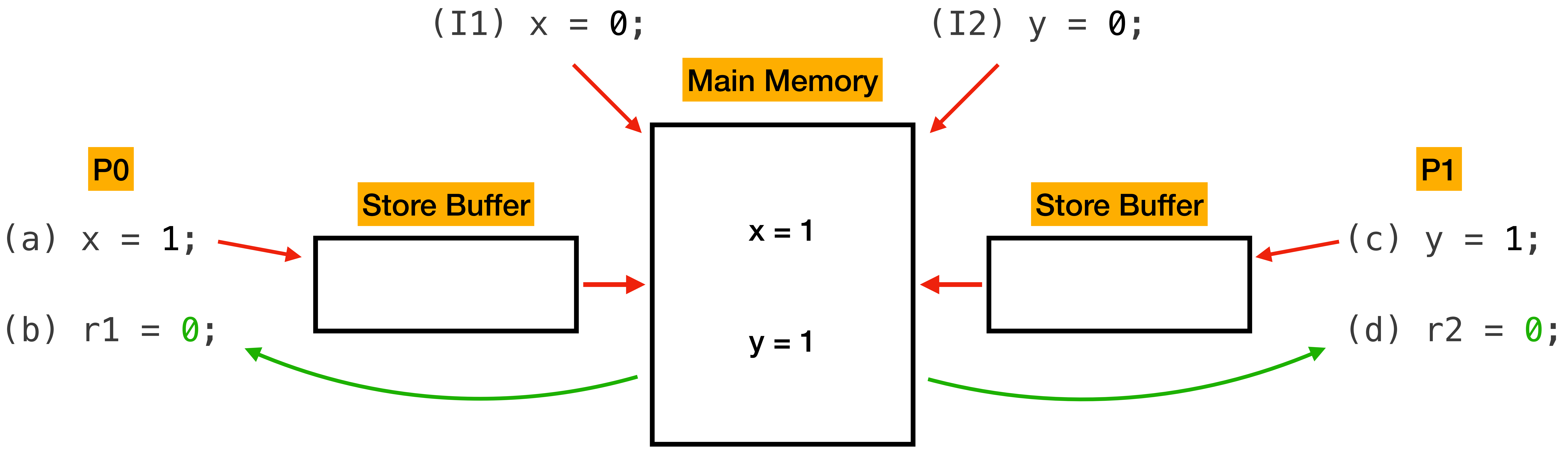
      Directly model observable behavior!
      - Axiomatic program/memory semantics!

Axiomatic program semantics

Example



Axiomatic program semantics

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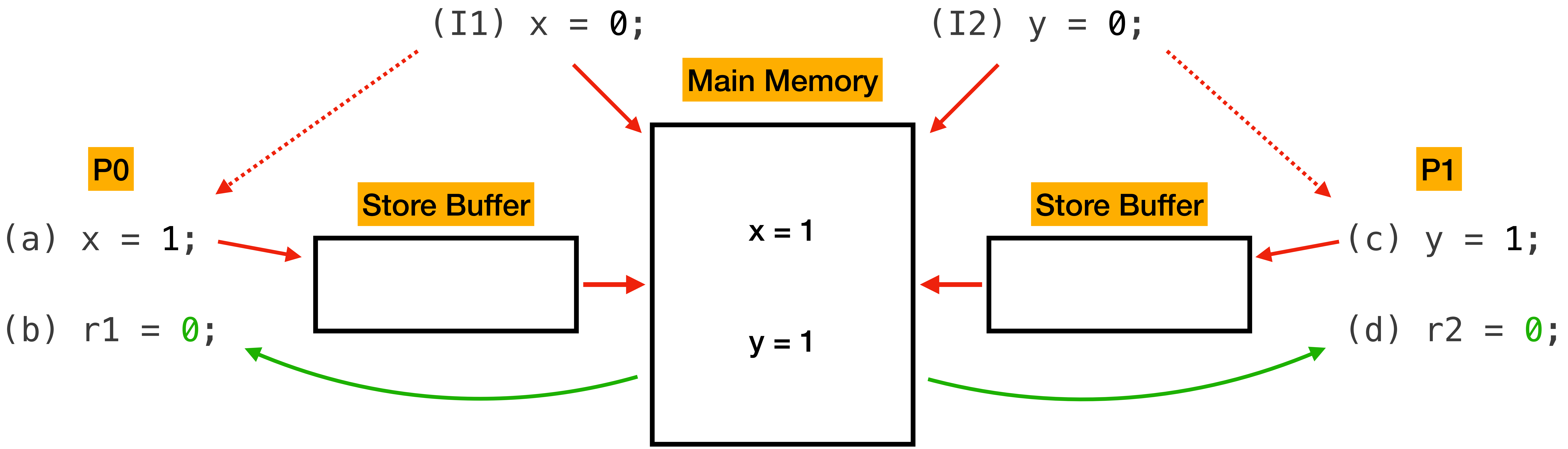

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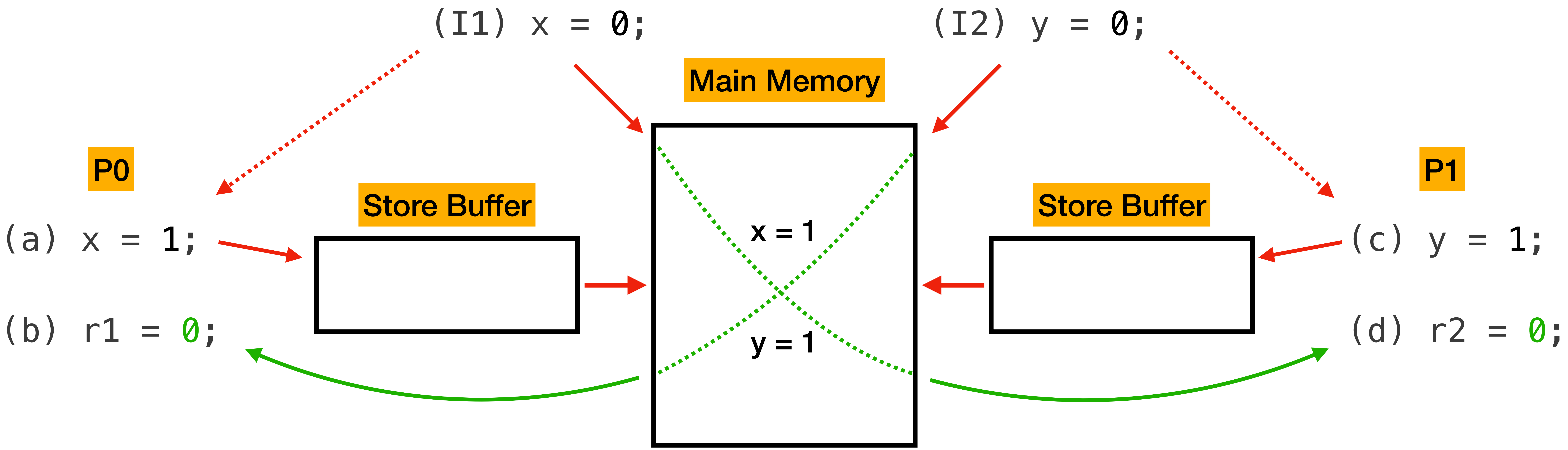

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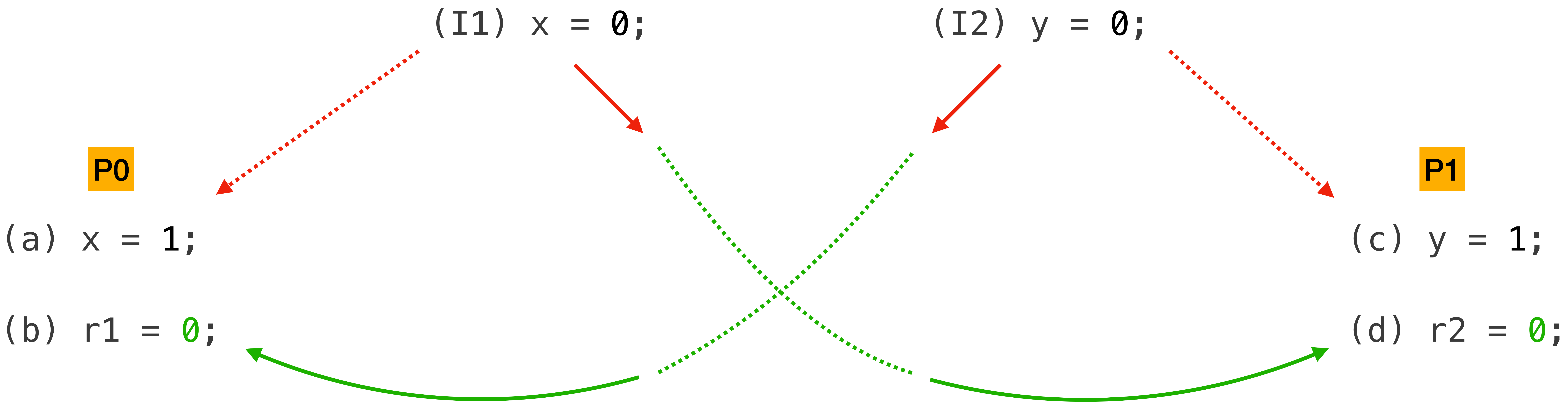

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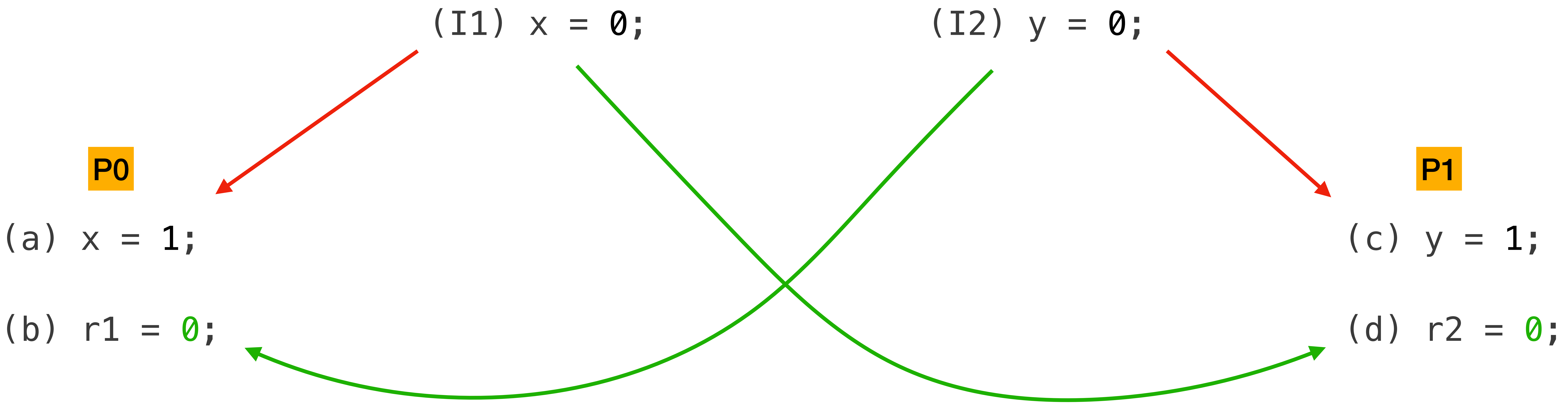

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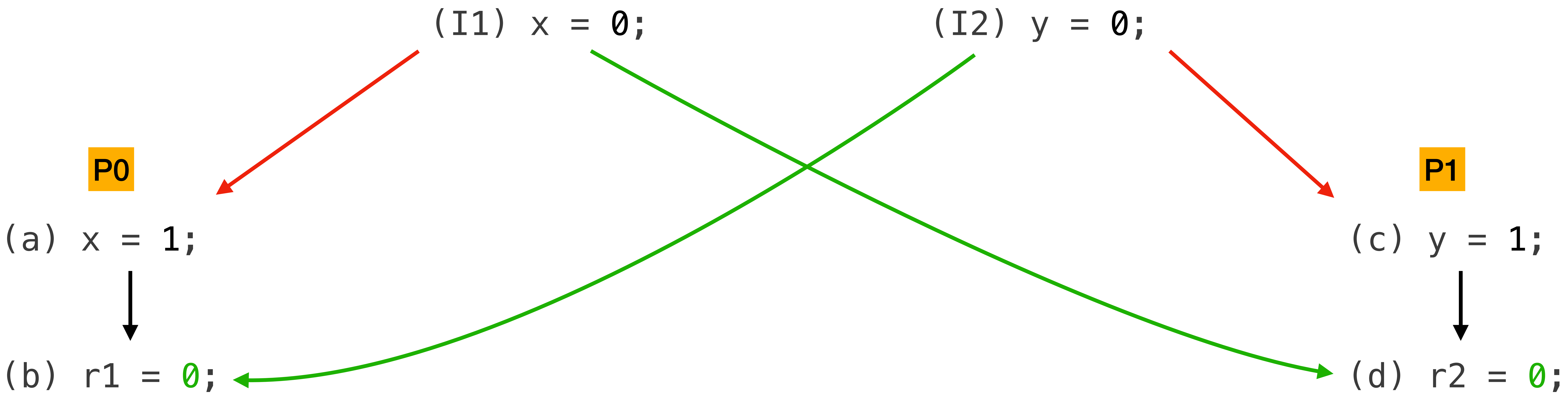

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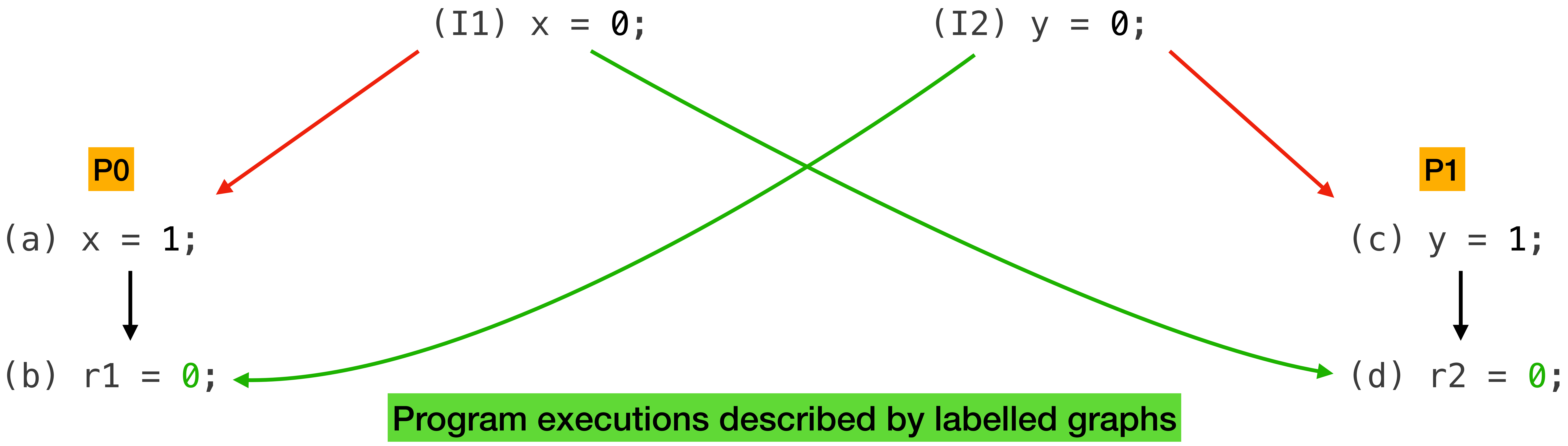

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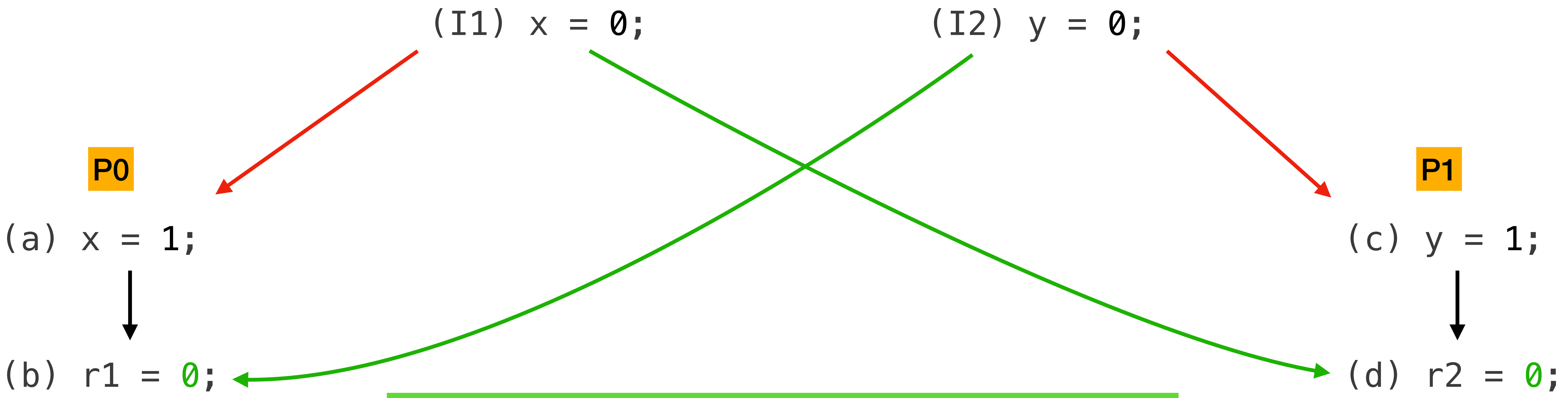

Program executions described by labelled graphs

No more interleavings!

Axiomatic program semantics

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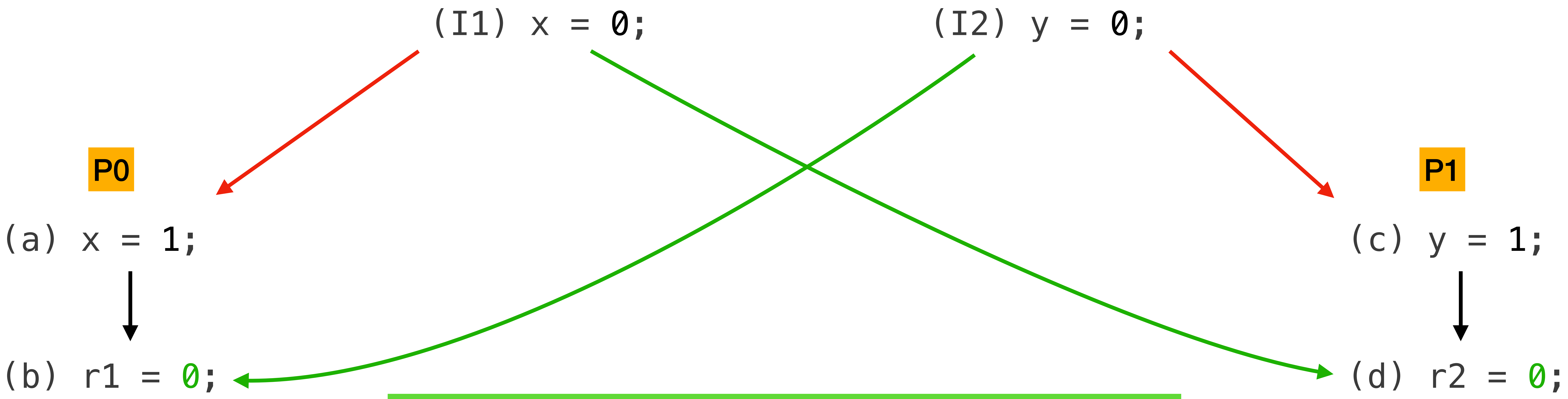

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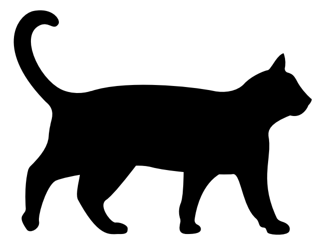

Consistency models with CAT*

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Industry Standard

Memory consistency models

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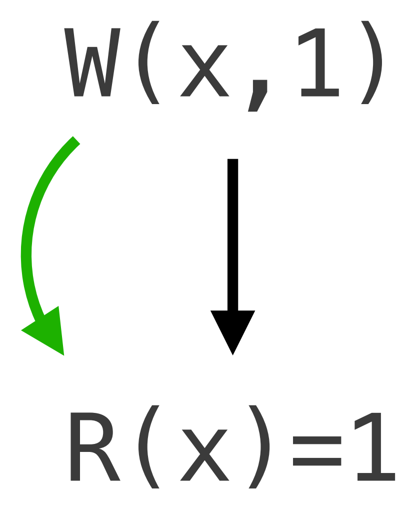

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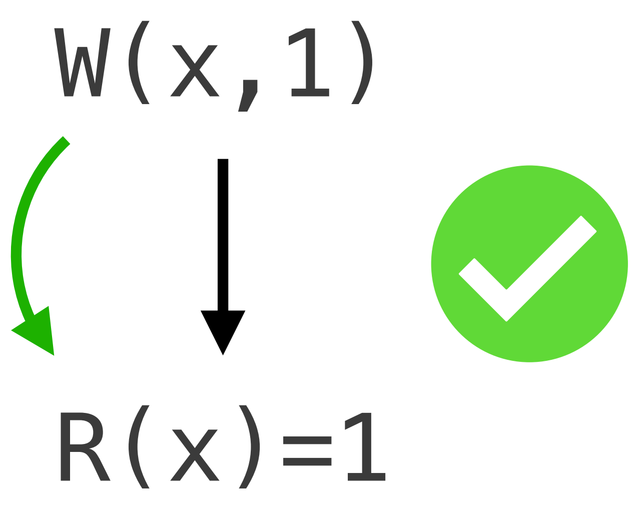

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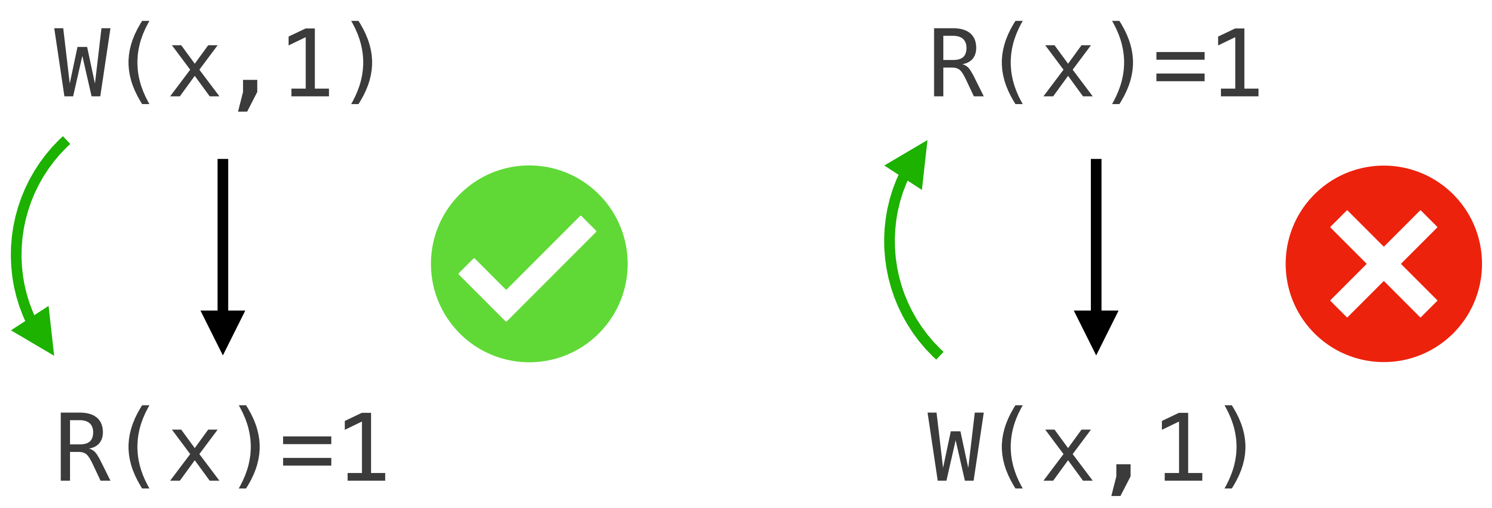

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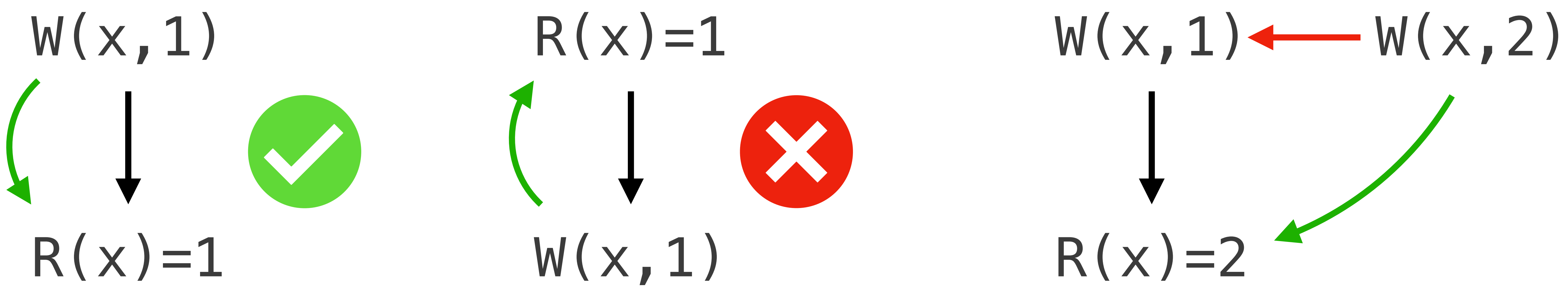

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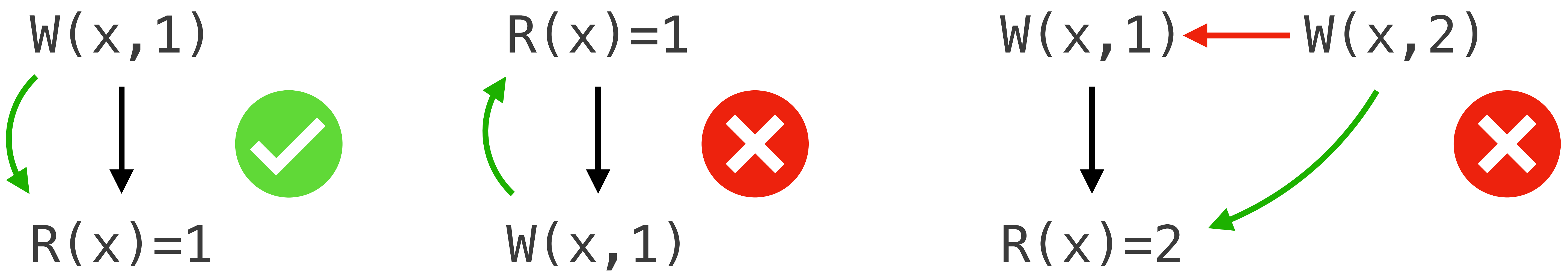

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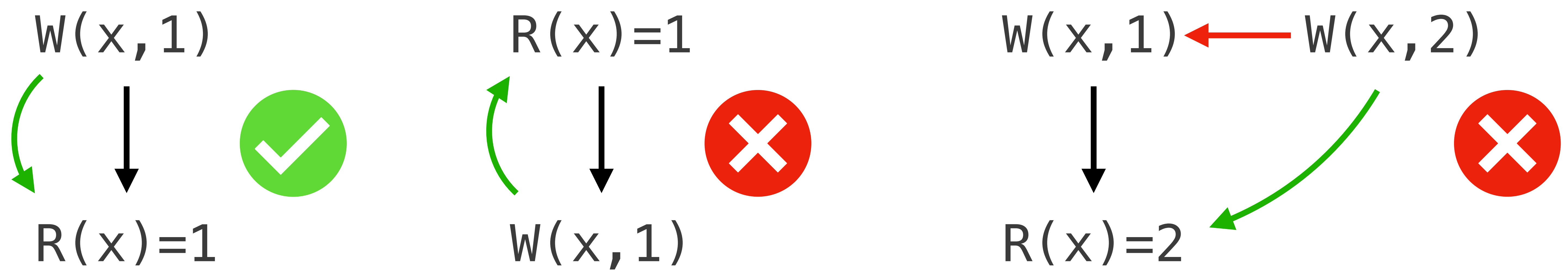

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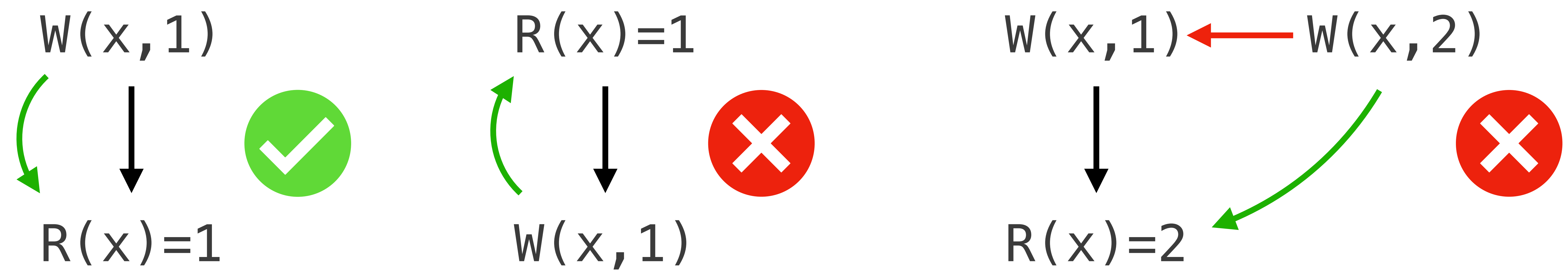

The CAT language is used to formulate memory consistency models

Restrict the shape (events & relations) of executions

Memory consistency models

The CAT language

CAT uses existing (base) relations

Memory consistency models

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Base relations

Memory consistency models

The CAT language

CAT uses existing (base) relations to define new ones (derived)

Base relations

- → Program order
- → Coherence order
- → Read-from relation
- → …

Memory consistency models

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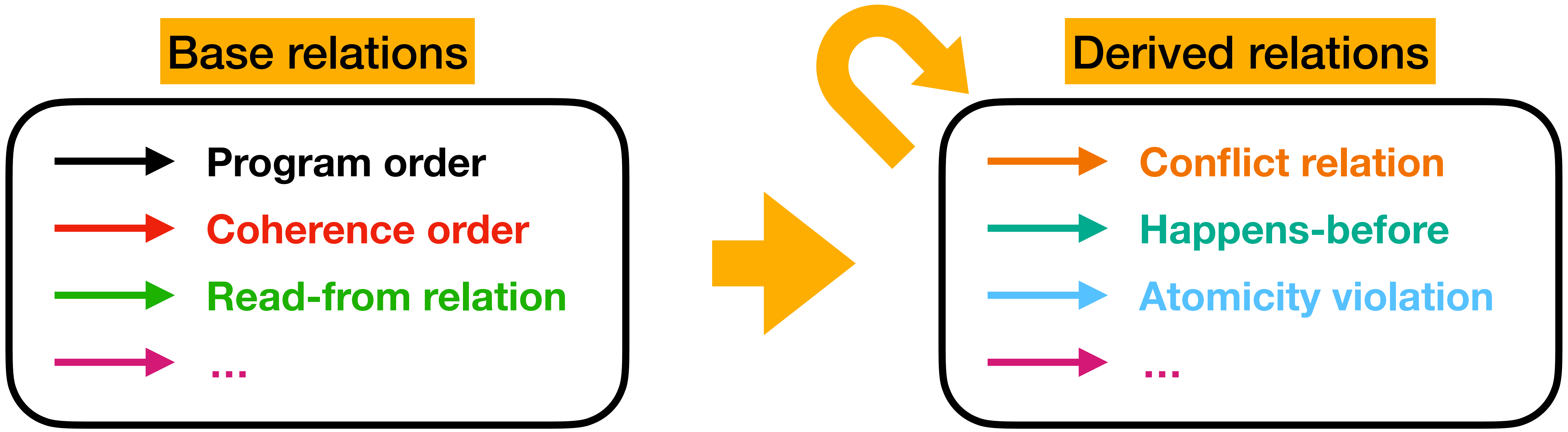

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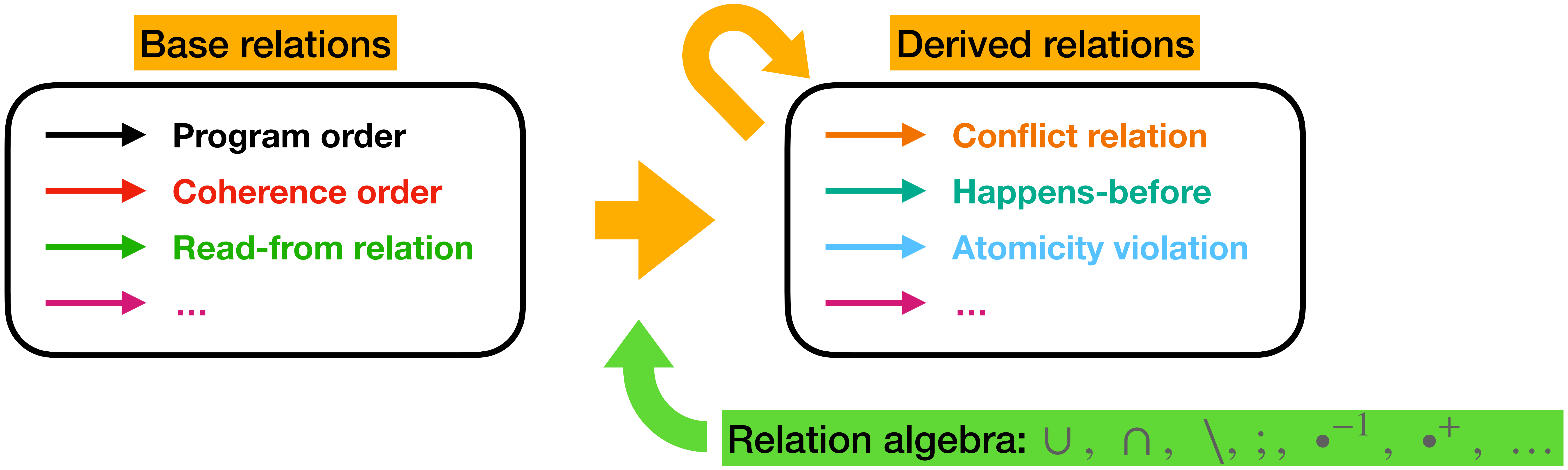

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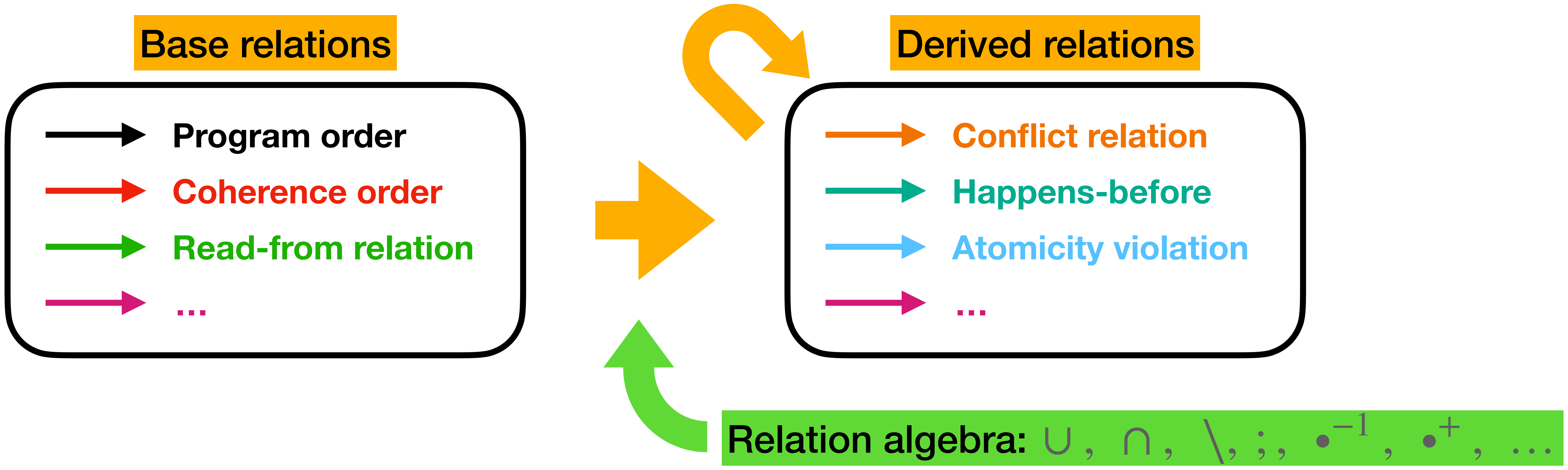
CAT puts constraints on relations, happens-before has to be acyclic



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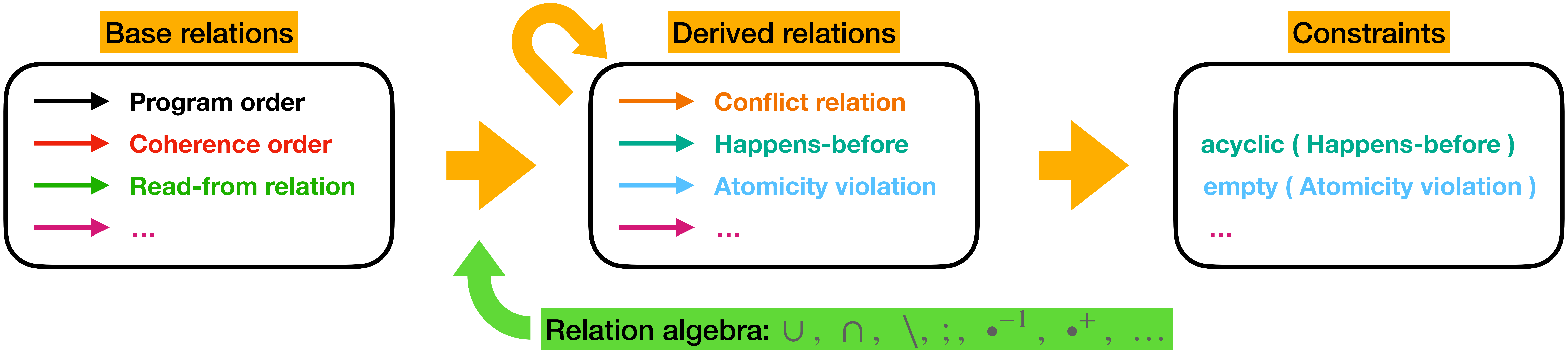

Memory consistency models

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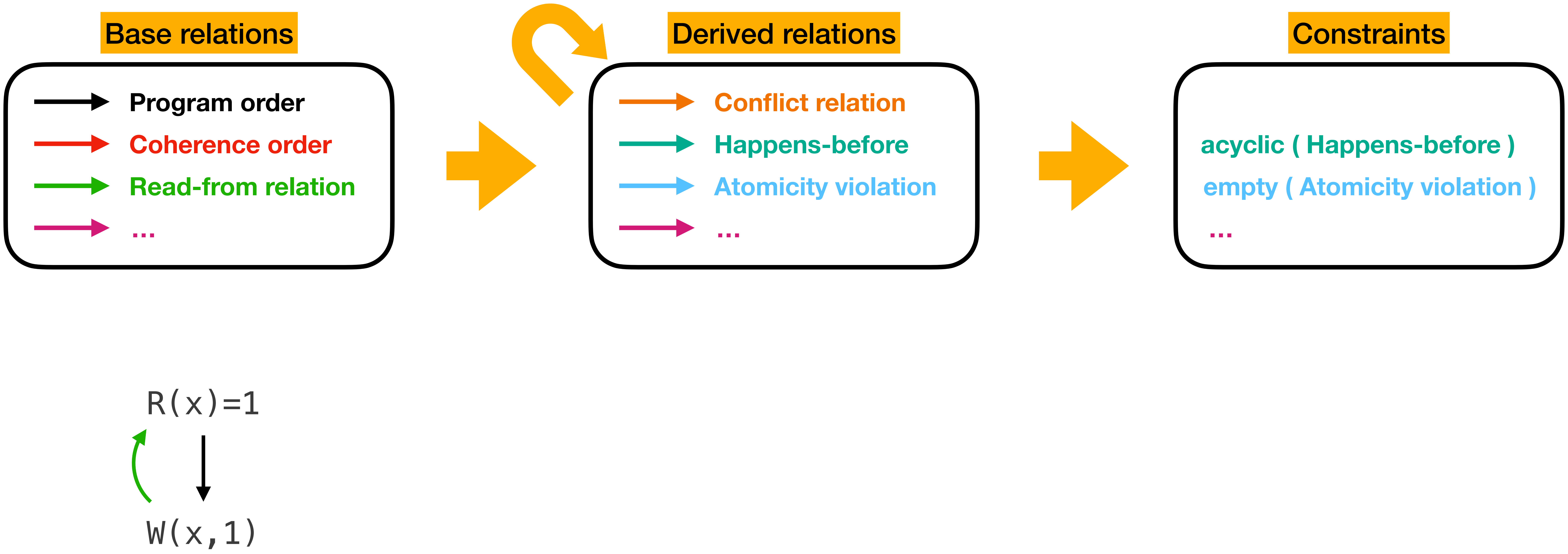

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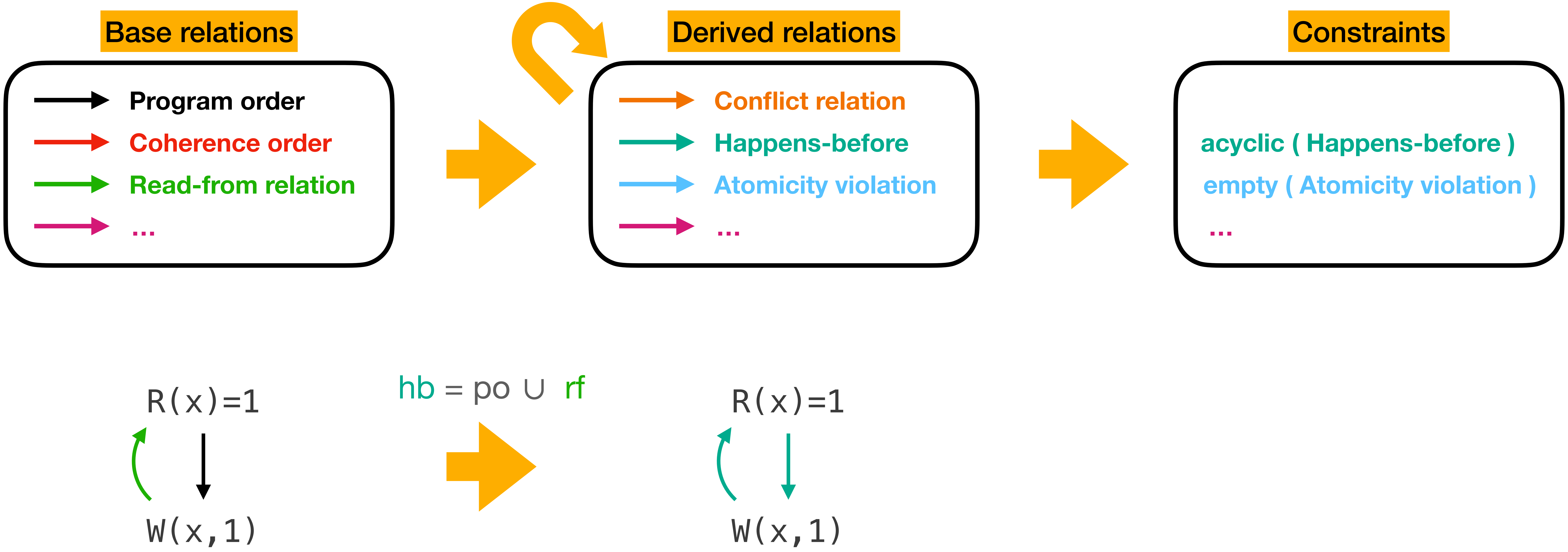

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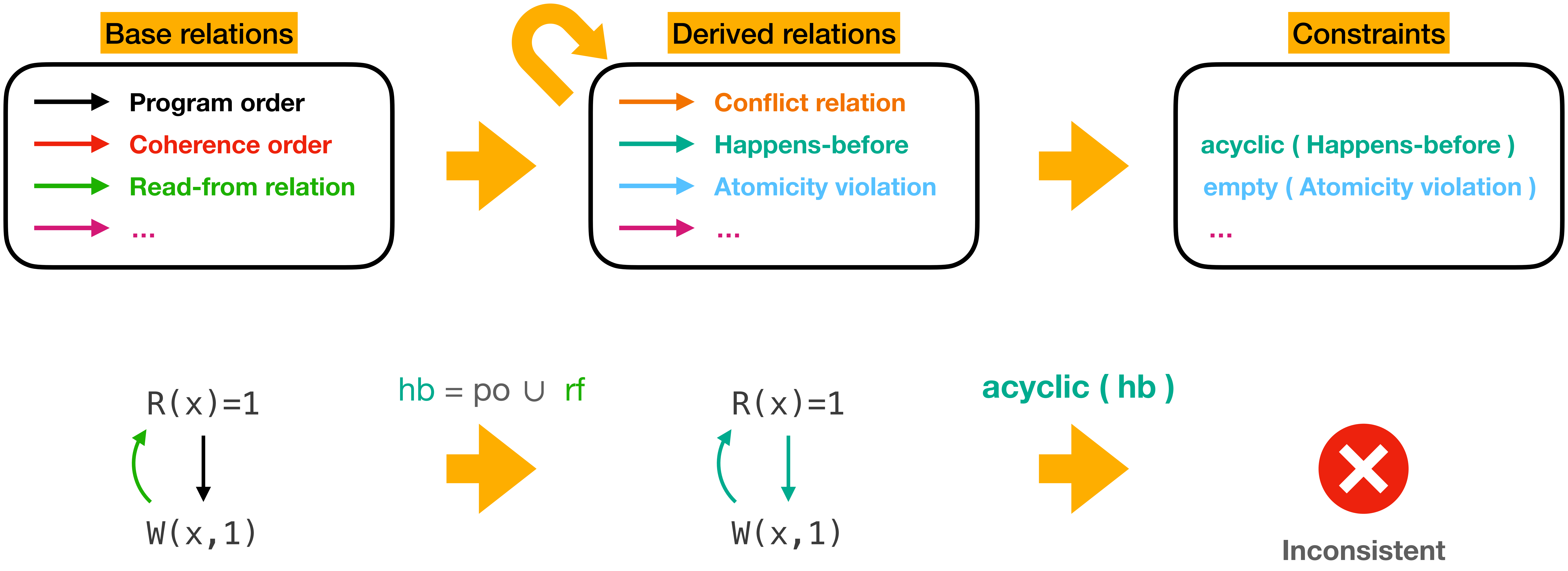

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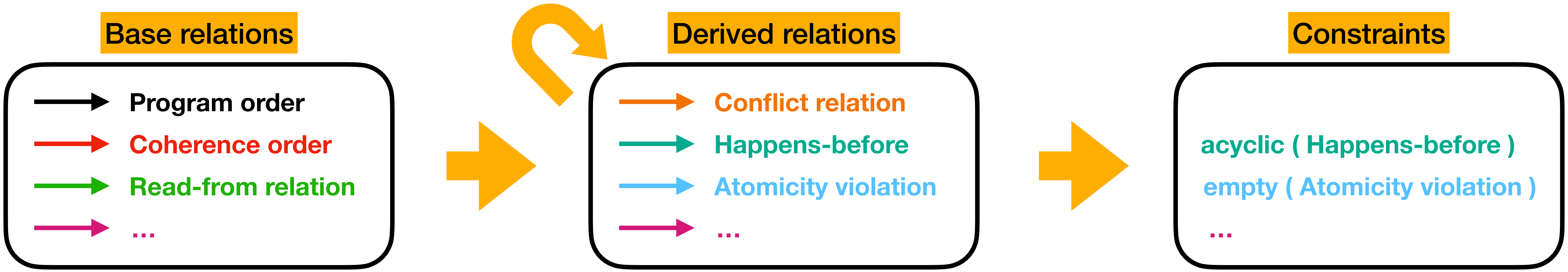

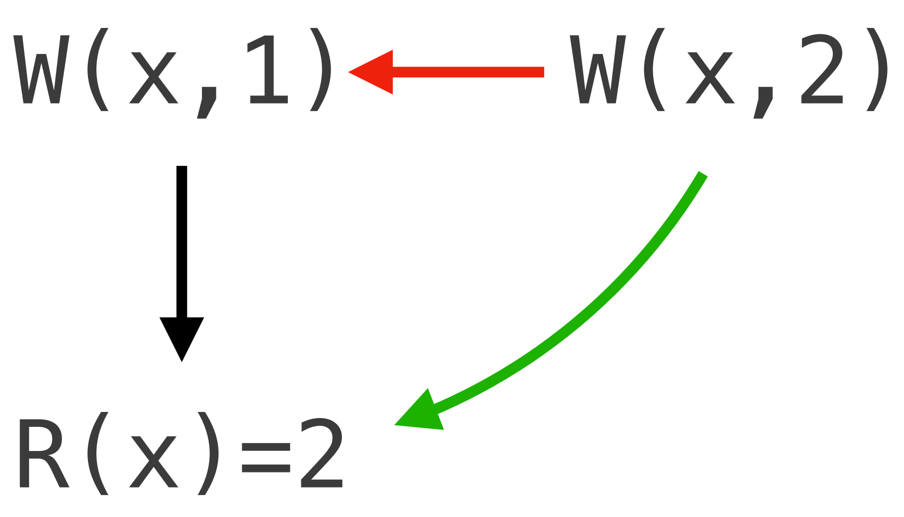

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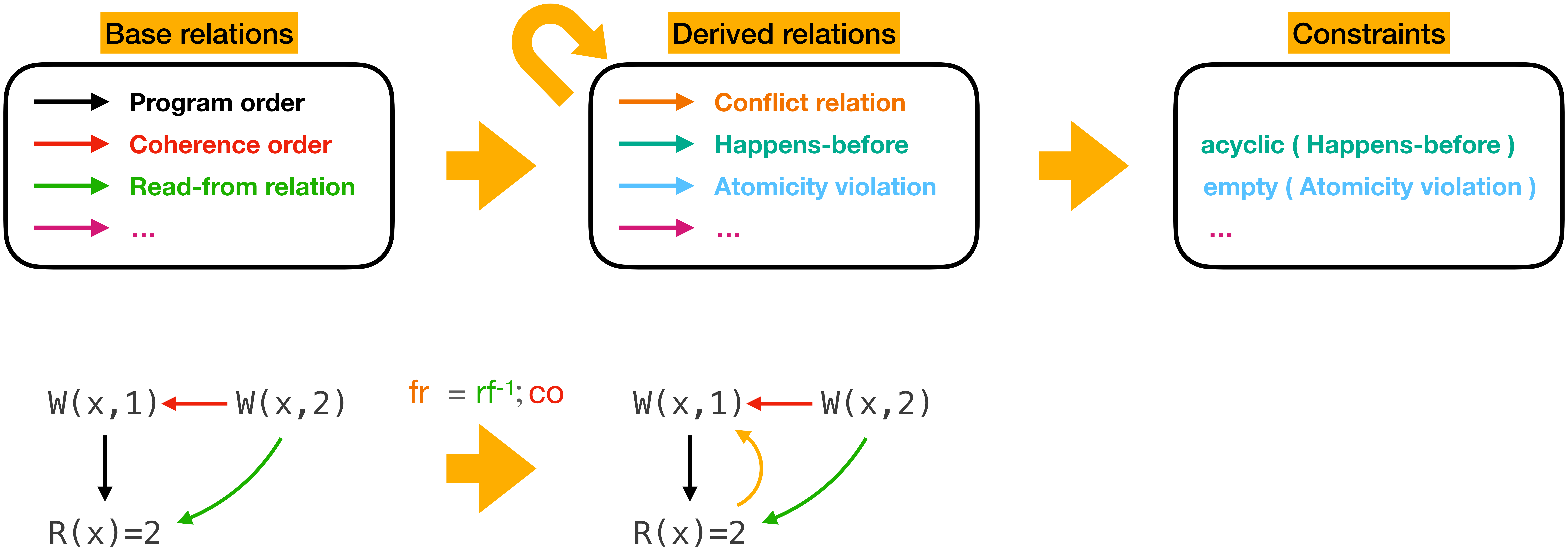

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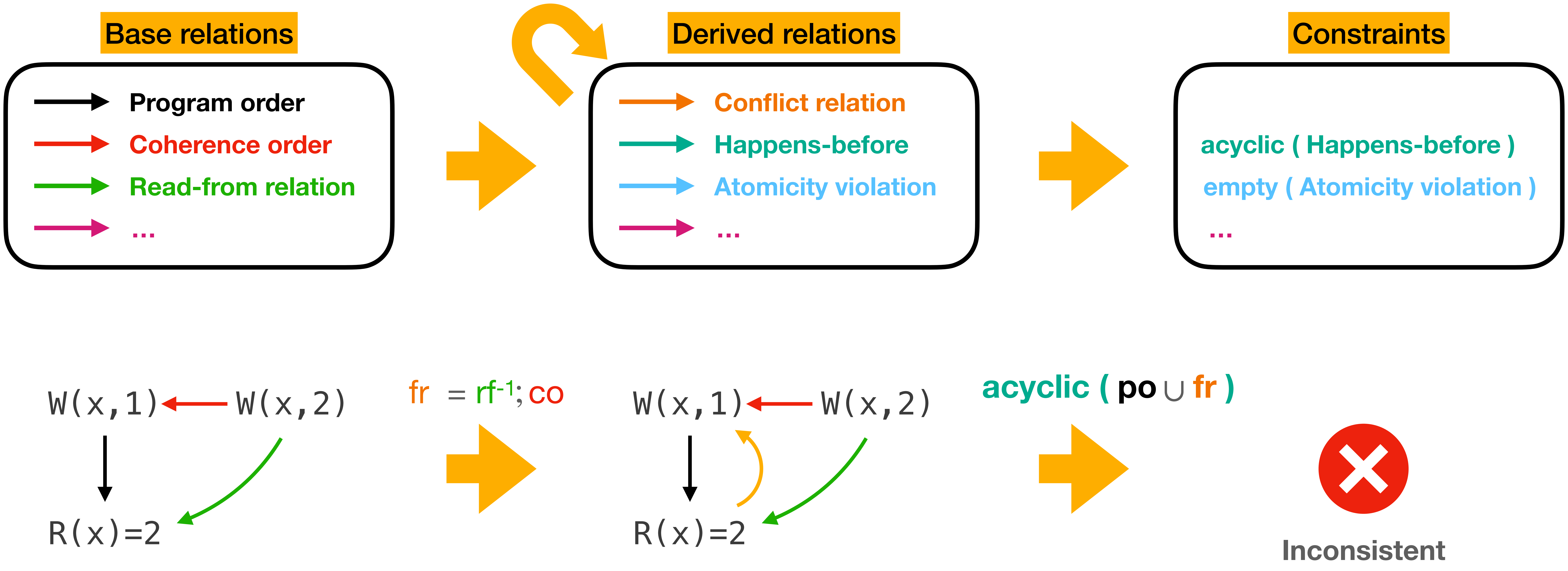

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Outlook

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Beyond hardware memory architectures!

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(A) Language-level memory models (C11, LKMM, Java, …)

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(A) Language-level memory models (C11, LKMM, Java, …)

Compiler optimisations + compiler mappings

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Beyond hardware memory architectures!

(A) Language-level memory models (C11, LKMM, Java, …)

Compiler optimisations + compiler mappings

Library specifications: RCU, pthread, safe memory reclamation, …

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(B) Distributed systems (~ communication protocols)

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Compiler optimisations + compiler mappings

Library specifications: RCU, pthread, safe memory reclamation, …

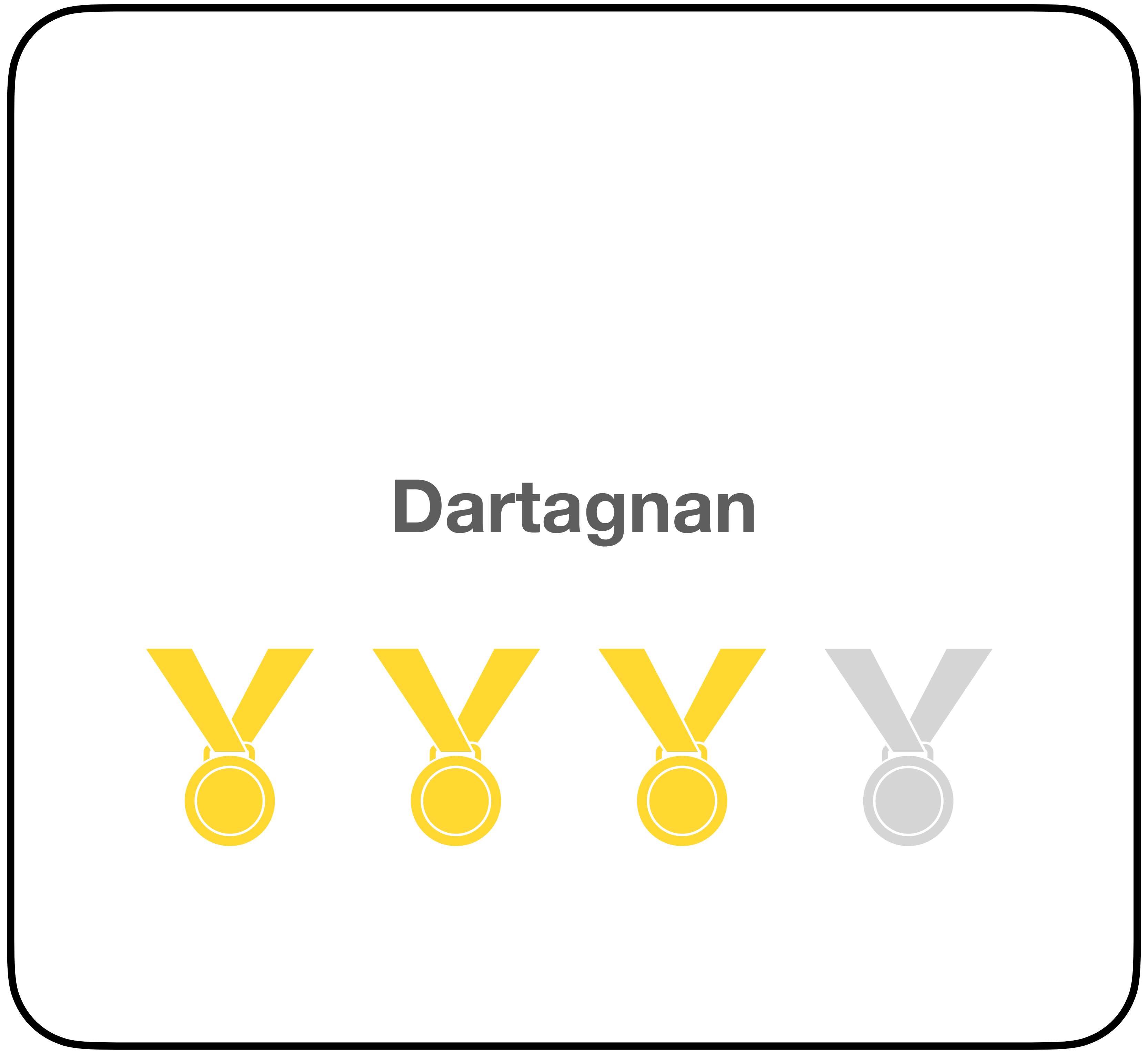
(B) Distributed systems (~ communication protocols)

(C) Databases (~ database isolation levels)

Dartagnan

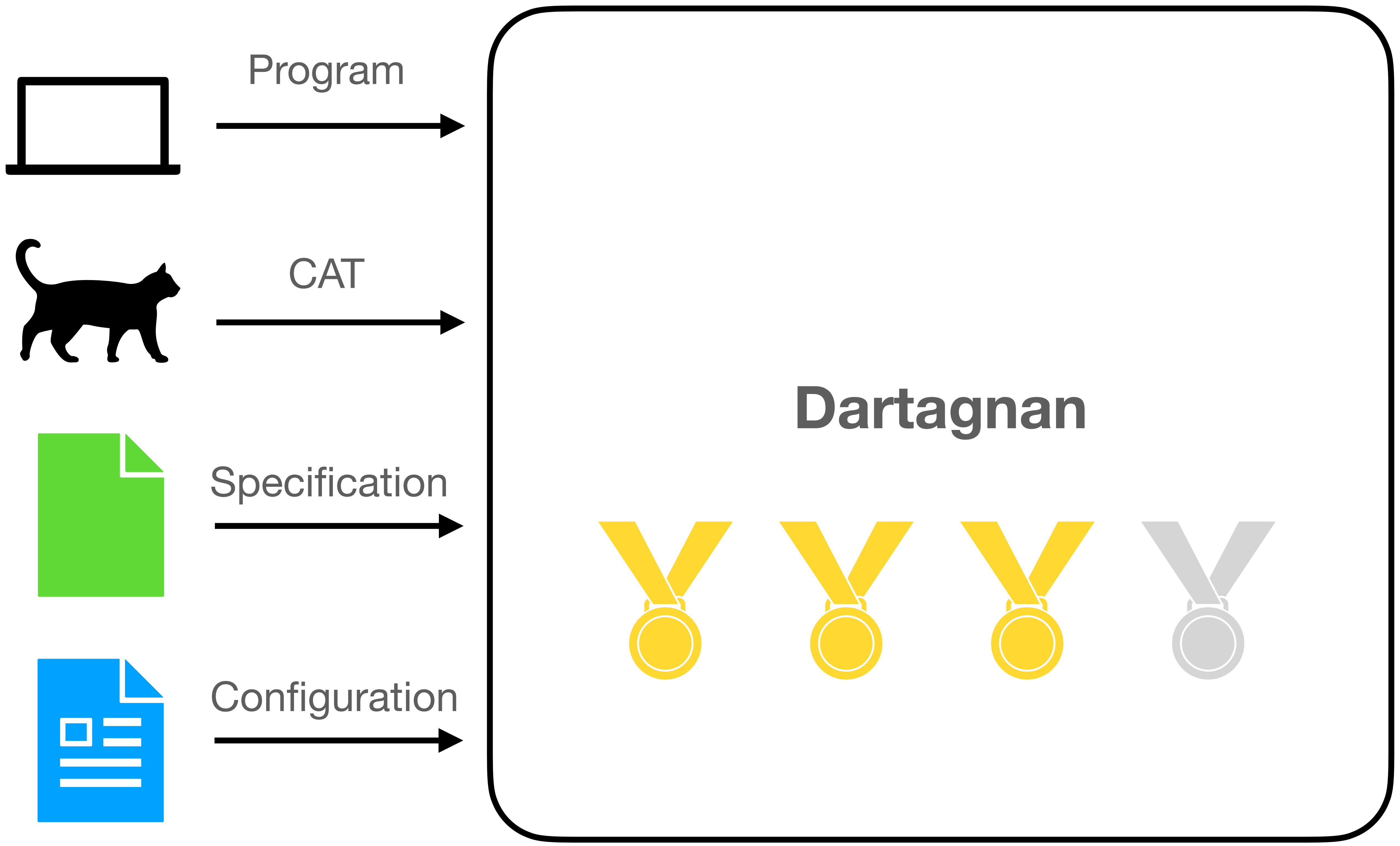
Dartagnan

Model checking real code



Dartagnan

Model checking real code

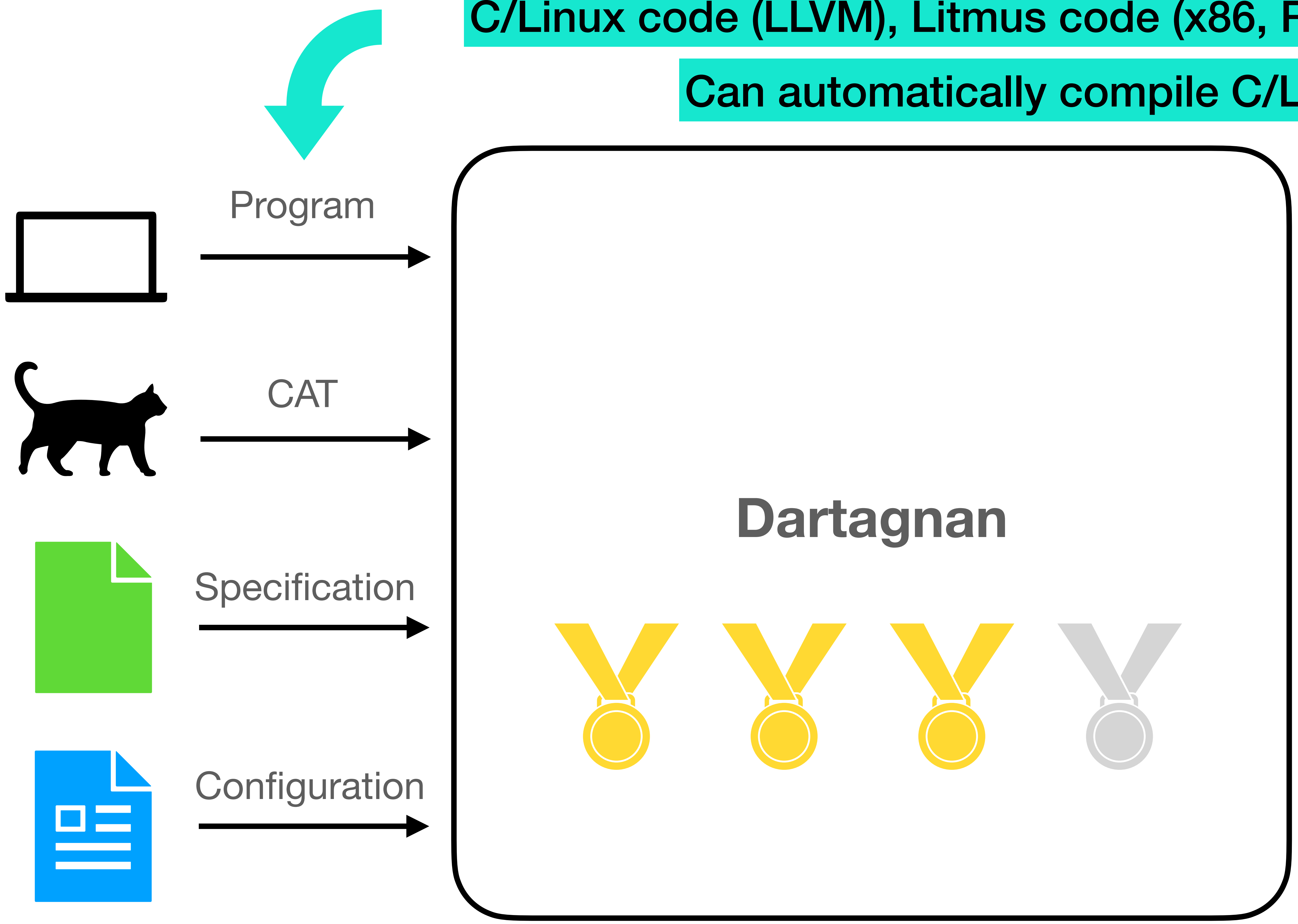


Dartagnan

Model checking real code

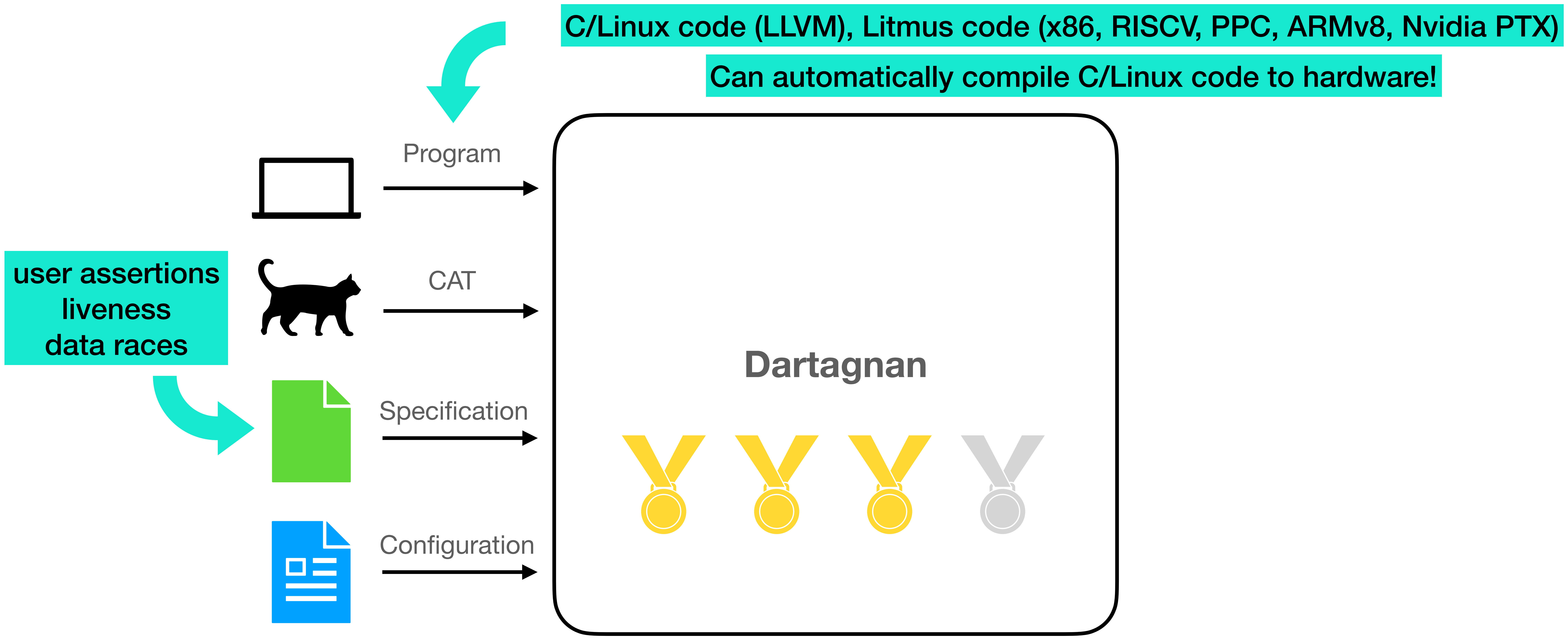
C/Linux code (LLVM), Litmus code (x86, RISCV, PPC, ARMv8, Nvidia PTX)

Can automatically compile C/Linux code to hardware!



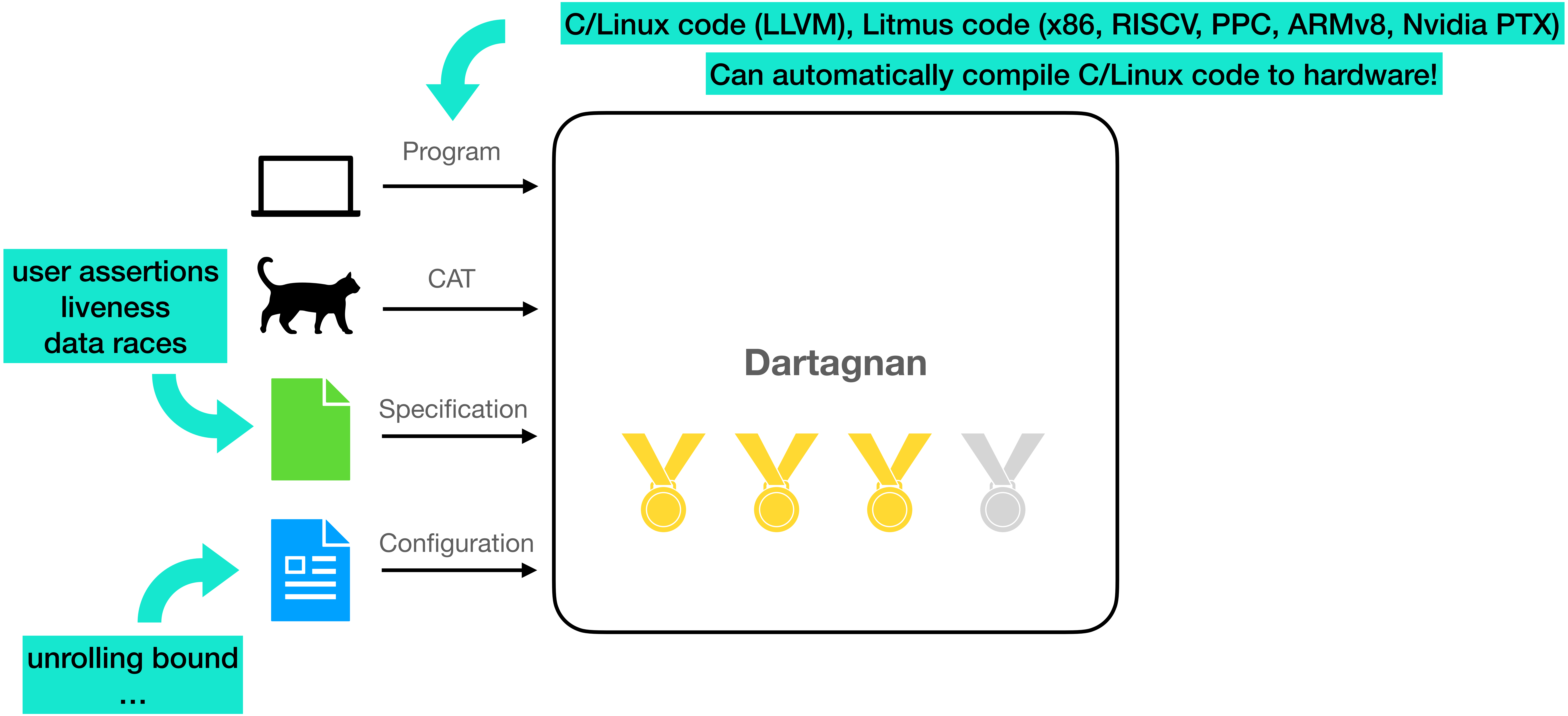
Dartagnan

Model checking real code



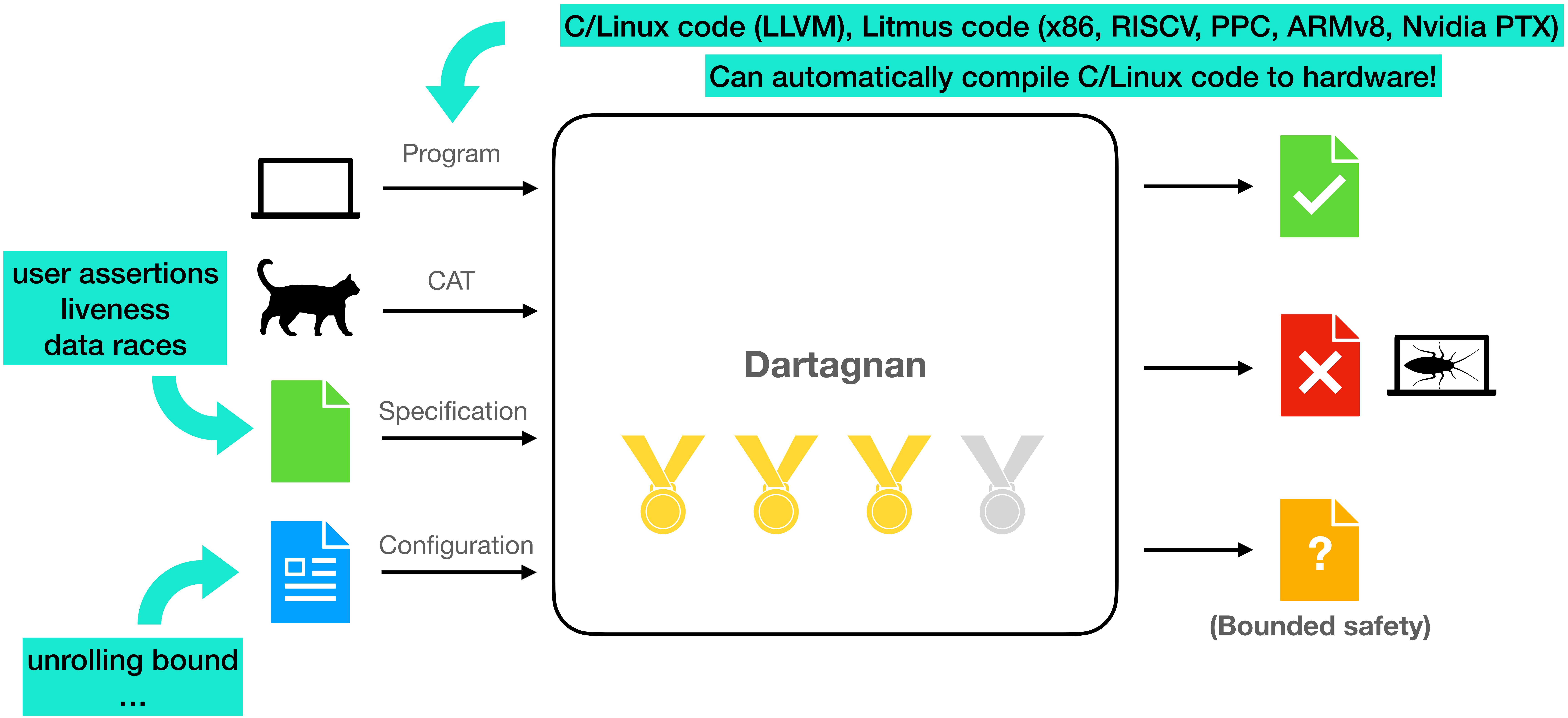
Dartagnan

Model checking real code



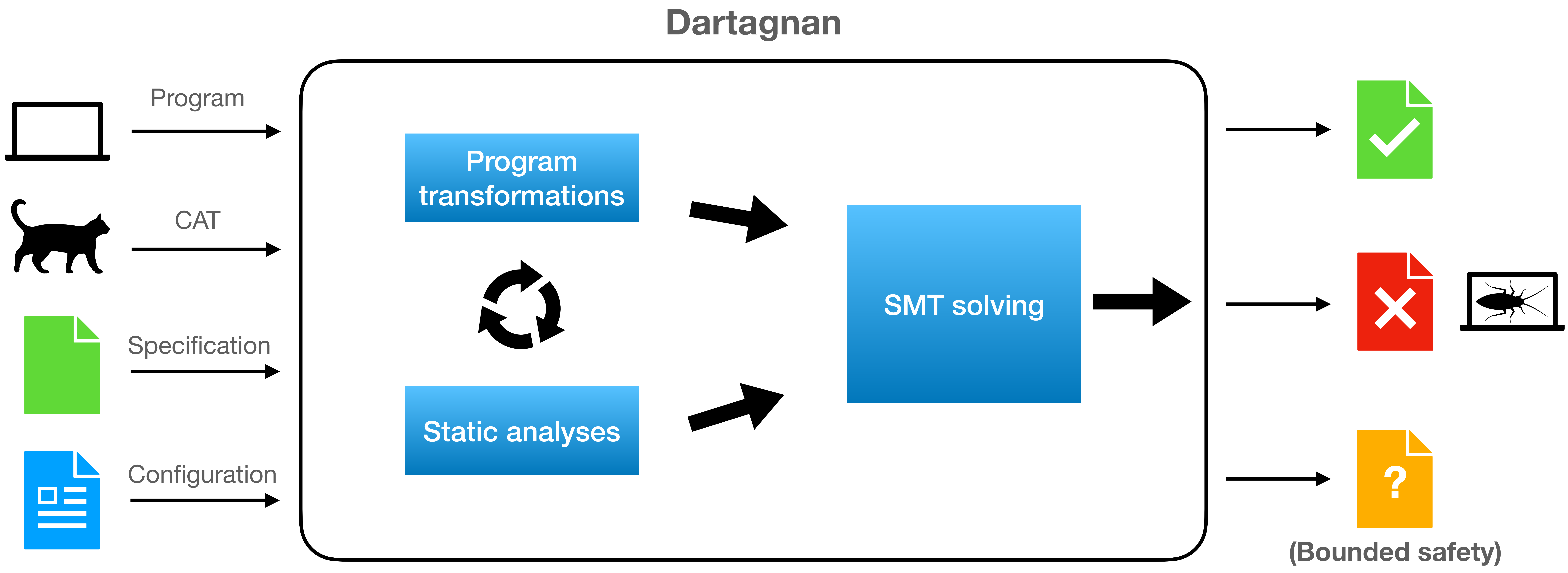
Dartagnan

Model checking real code

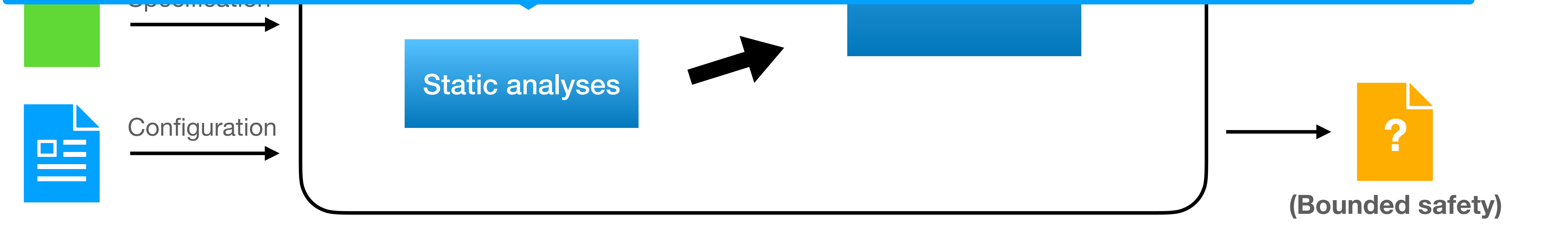


Dartagnan

Internals

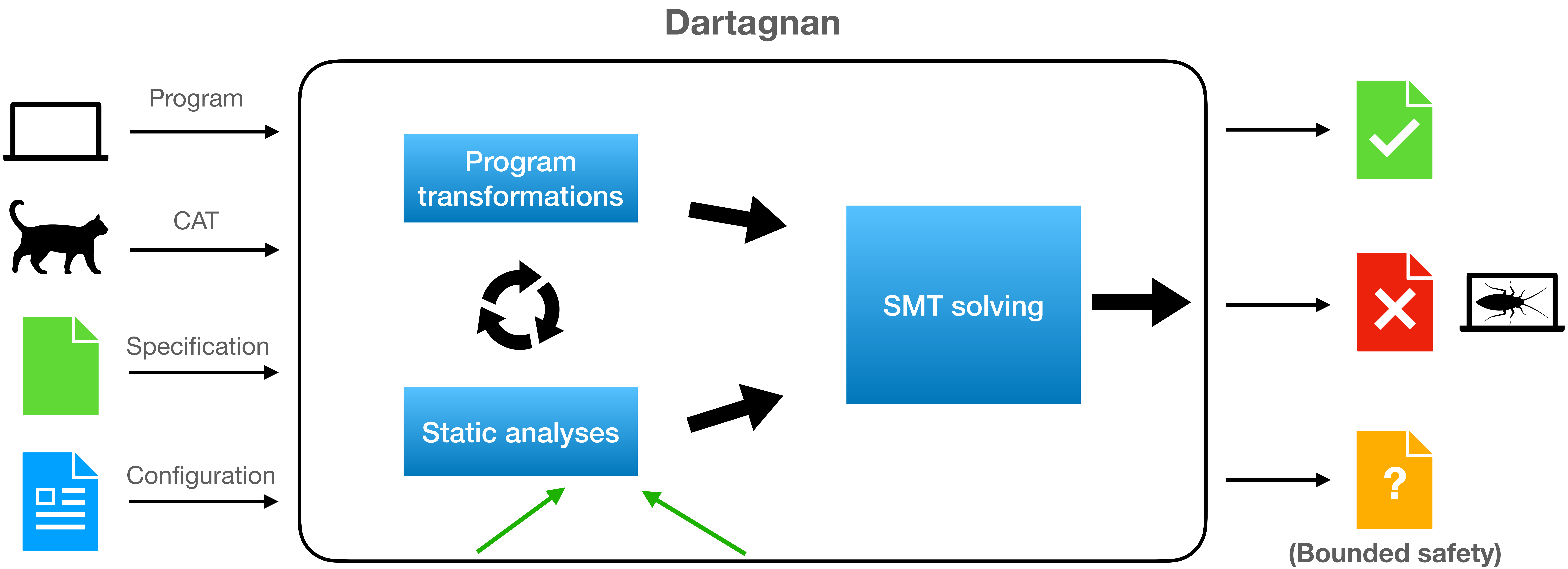


Alias Analysis
Control-Flow Analysis (find basic blocks of instructions executed together, control-flow variables)
Constant Propagation
Def-Use-Analysis
Dominator Analysis
Expression Simplification
Function Call Devirtualization (resolve call targets)
Function Inlining
Live Variables
Loop Unrolling
Mem2Reg (treat stack as registers)
Normalize Loops (single backjump, single entry)
Reaching Definitions
Sparse Conditional Constant Propagation (constant propagation + dead code elimination)
Spin Loop Detection and Instrumentation for Dynamic Detection
Symmetry Breaking



Dartagnan

Internals



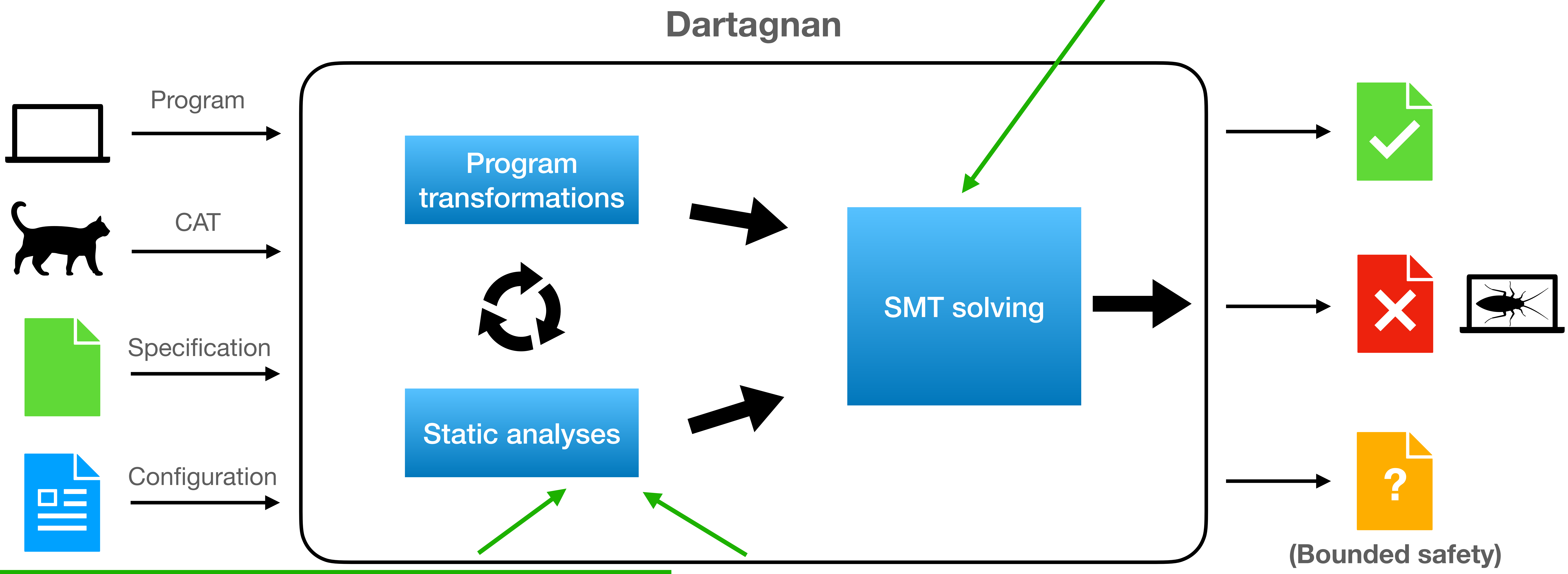
Natalia Gavrilenko, Hernán Ponce de León, Florian Furbach, Keijo Heljanko, Roland Meyer: *BMC for Weak Memory Models: Relation Analysis for Compact SMT Encodings @ CAV19*

Thomas Haas, René Maseli, Roland Meyer, Hernán Ponce de León: *Static Analysis of Memory Models for SMT Encodings @ OOPSLA23*

Dartagnan

Internals

Thomas Haas, Roland Meyer, Hernán Ponce de León: *CAAT: consistency as a theory @ OOPSLA22*



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Encoding CAT into logical theories

Thomas Haas, Roland Meyer, Hernán Ponce de León:
CAAT: consistency as a theory @ OOPSLA22

CAT in logical theories

CAT in logical theories

- CAT has simple operations over relations: $;\,,\cup\,,\cap\,,\setminus,\bullet^{-1}$

  $\rightarrow$ Easily encodable into plain SAT (over finite domain)

CAT in logical theories

- CAT has simple operations over relations: $;\,,\cup\,,\cap\,,\backslash,\bullet^{-1}$

  → Easily encodable into plain SAT (over finite domain)

- CAT has axioms on relations: **empty**, **irreflexive**, **acyclic**

  → Emptiness and irreflexivity encodable into plain SAT;
  Acyclicity encodable into integer difference logic (SMT)

CAT in logical theories

- CAT has

→ Easily

Problem: CAT allows for (non-linear) recursive definitions with (stratified) least fixed point semantics!

- CAT has axioms on relations: **empty**, **irreflexive**, **acyclic**

→ Emptiness and irreflexivity encodable into plain SAT; Acyclicity encodable into integer difference logic (SMT)

CAT in logical theories

- CAT has

→ Easily

Problem: CAT allows for (non-linear) recursive definitions with (stratified) least fixed point semantics!

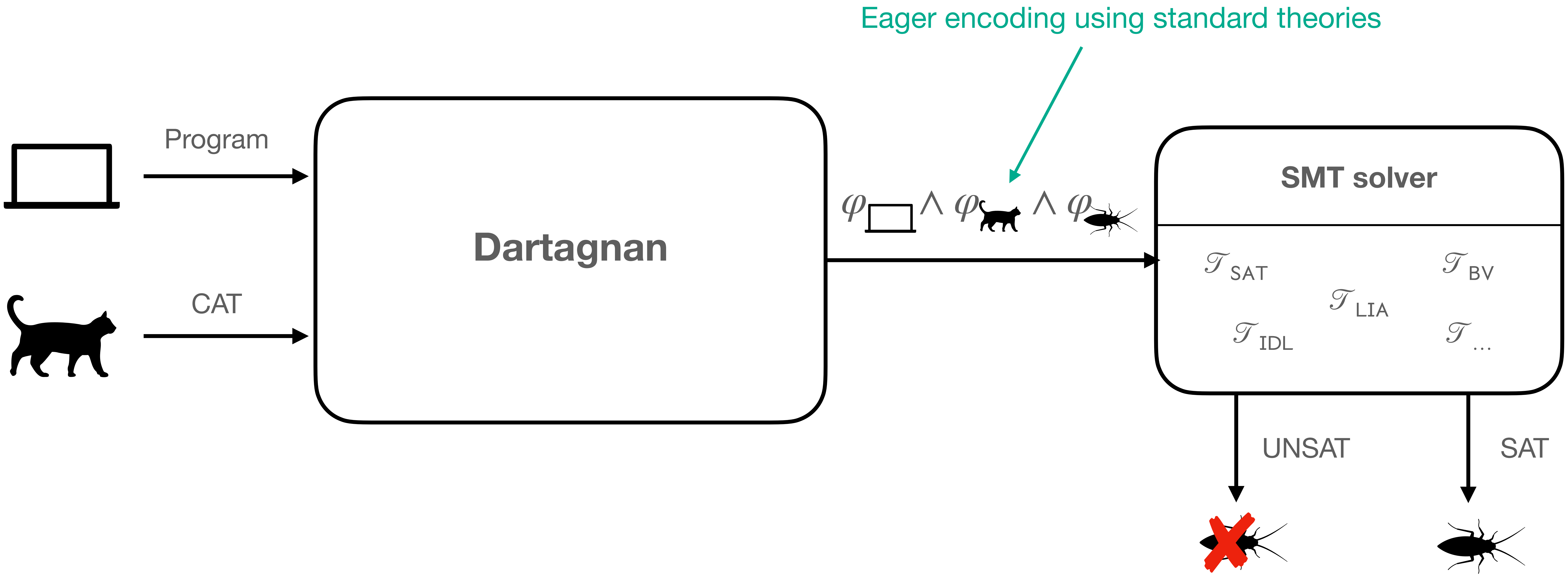
- C

Existing theories have a hard time capturing least fixed point semantics!

→ Emptiness and irreflexivity encodable into plain SAT; Acyclicity encodable into integer difference logic (SMT)

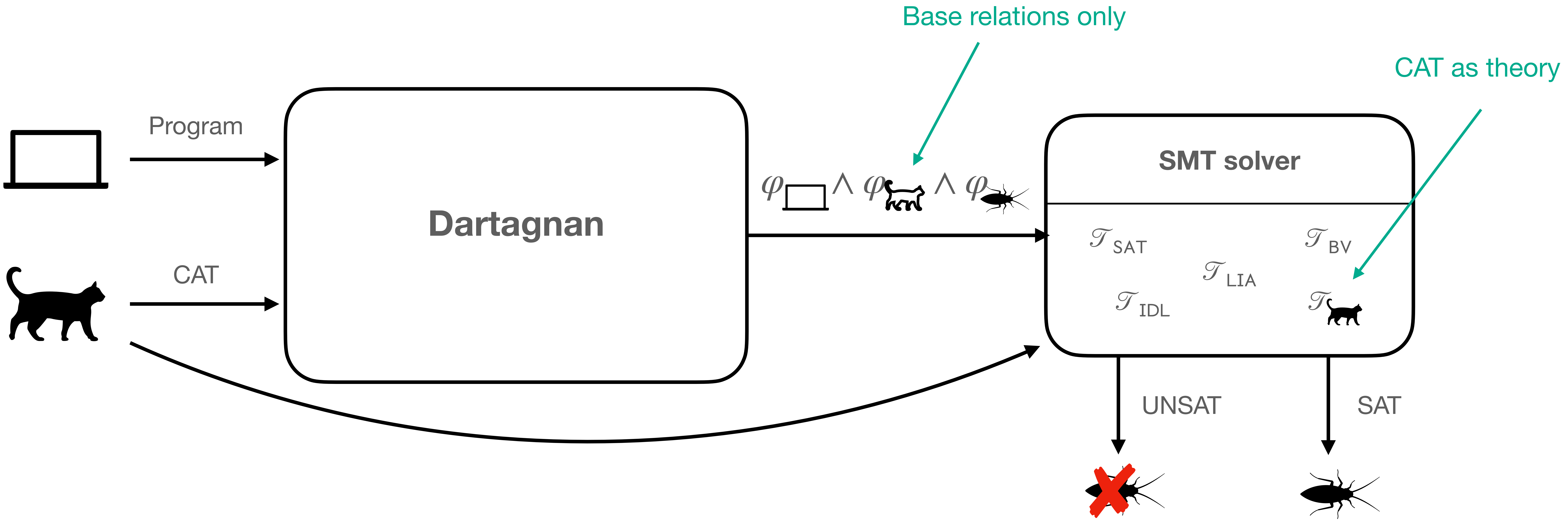
Dartagnan

Memory-model-parametric BMC



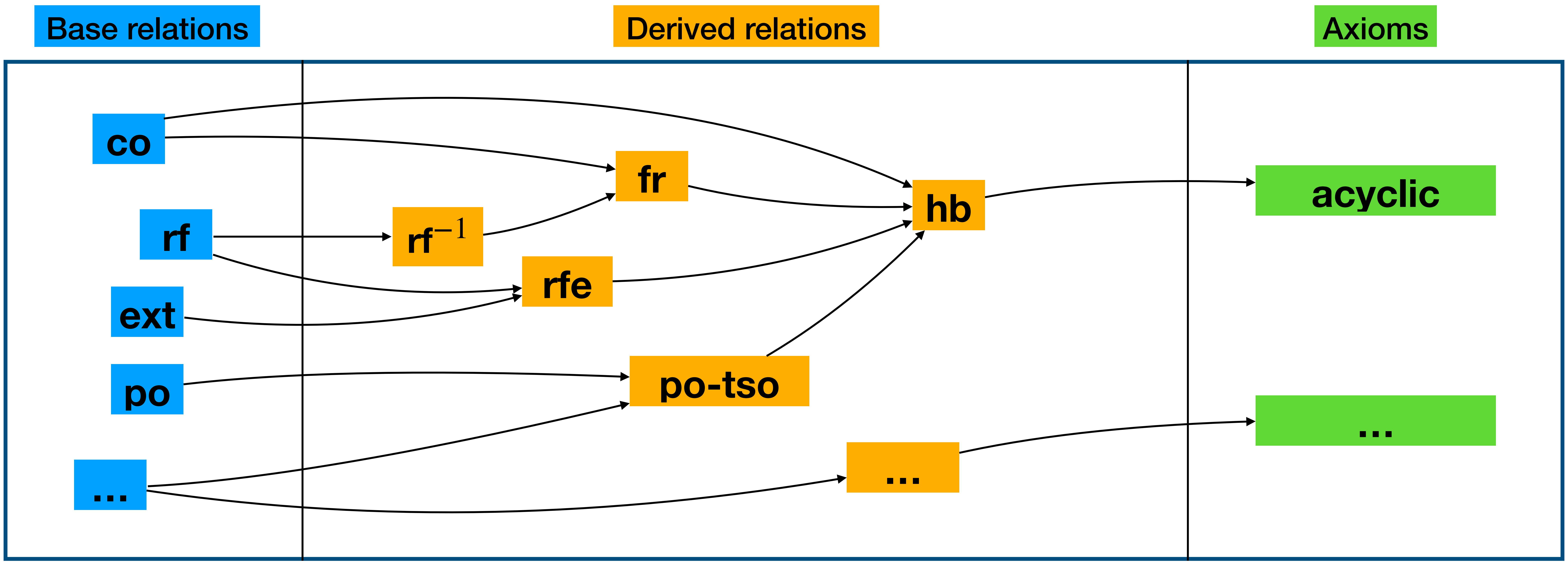
Dartagnan + CAAT

Memory-model-parametric BMC with CAAT

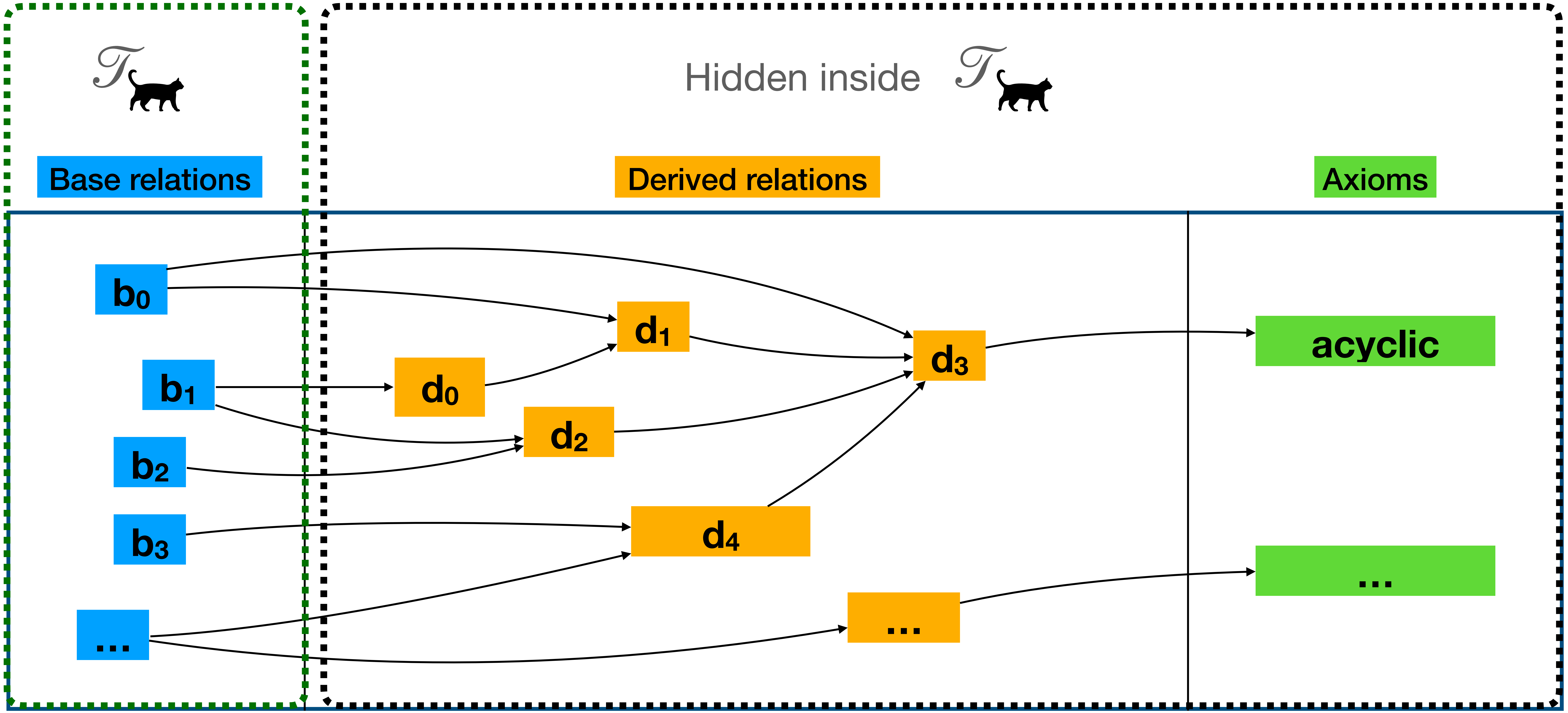


CAT as logical theory

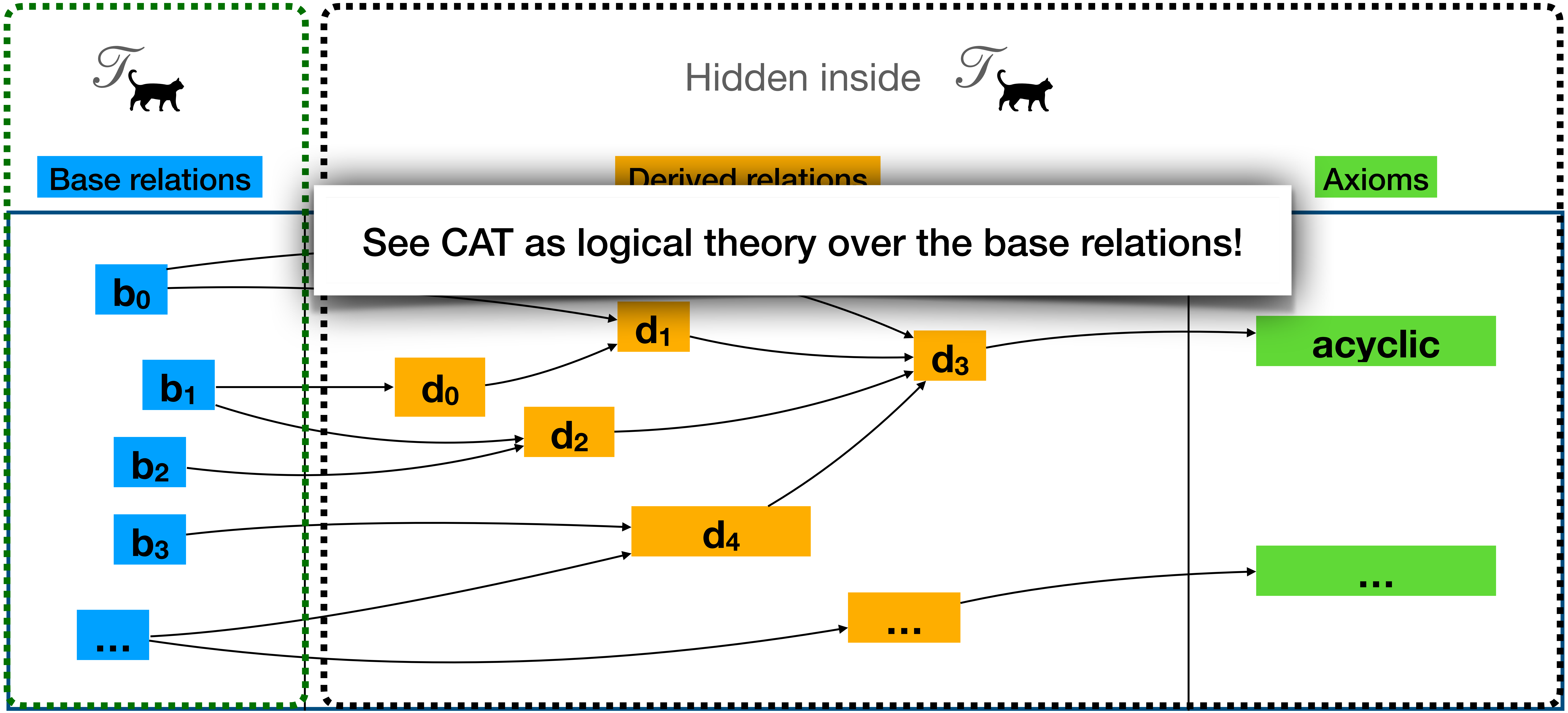
```
let fr = rf^-1;co
let po-tso = (po \ WxR) | mfence
let hb = po-tso | (rf & ext) | fr | co
acyclic hb
// more relations & axioms
```



CAAT: Consistency as a Theory



CAAT: Consistency as a Theory



See CAT as logical theory over the base relations!

How does it work?

Theory solving for SC

```
let fr = rf^-1;co
let hb = po | rf | fr | co
acyclic hb
```

Theory solving for SC

```
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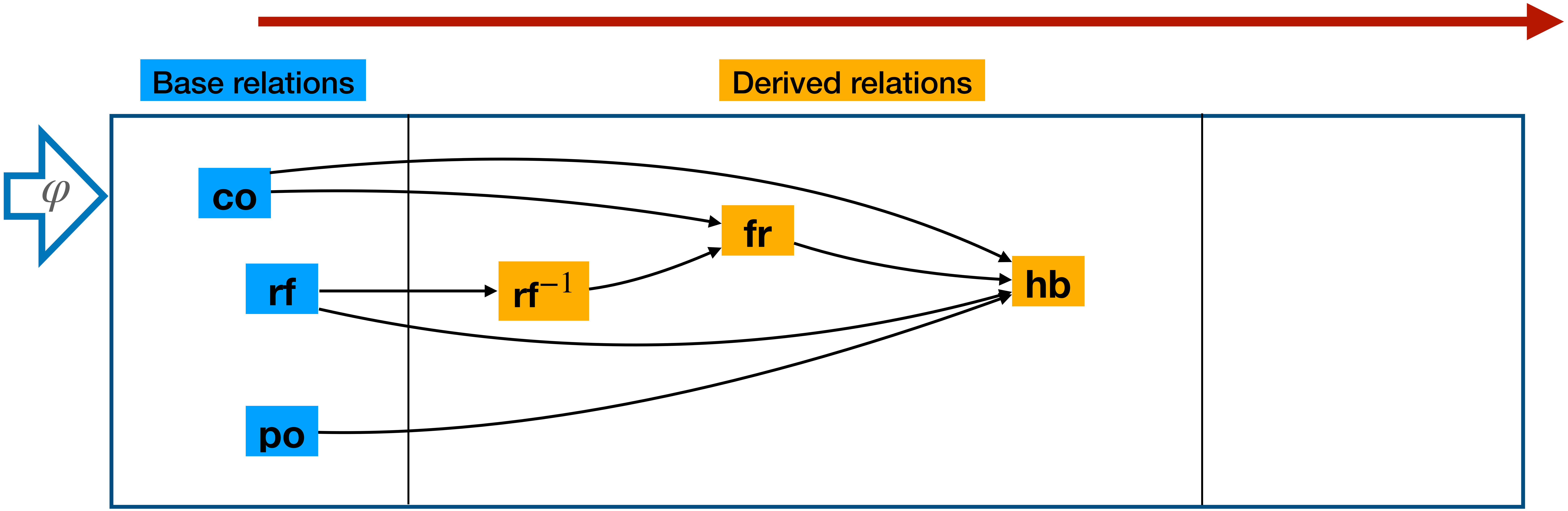
1. Derive (Bottom-Up)



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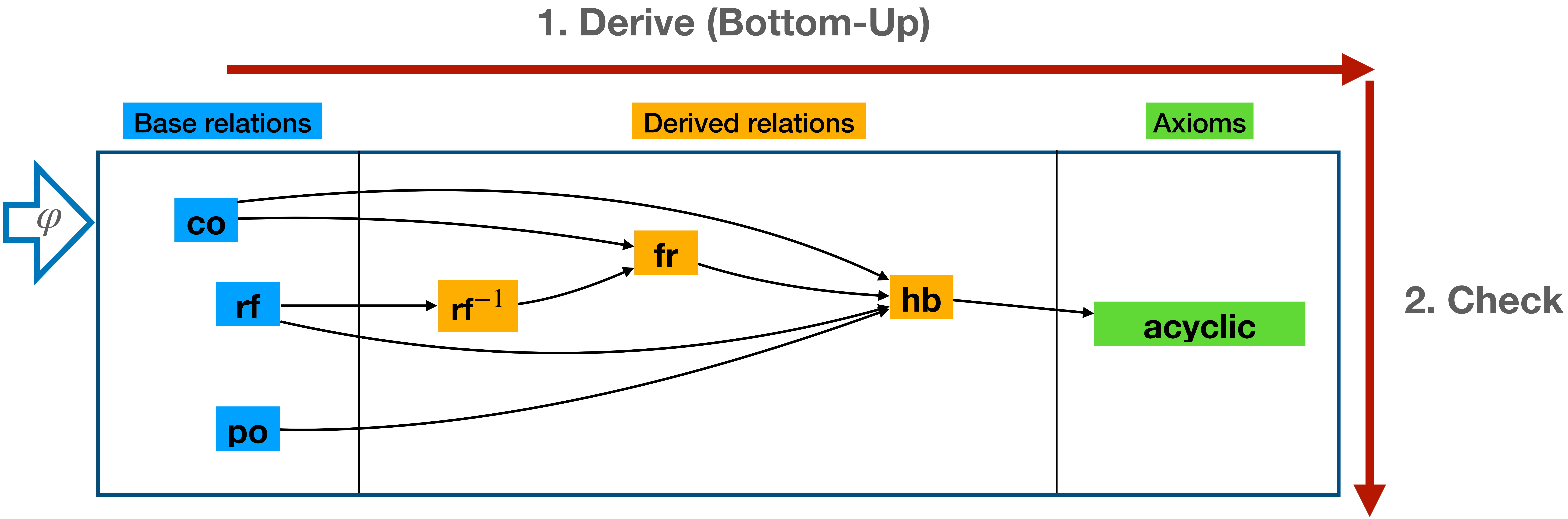

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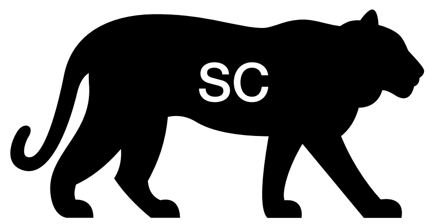


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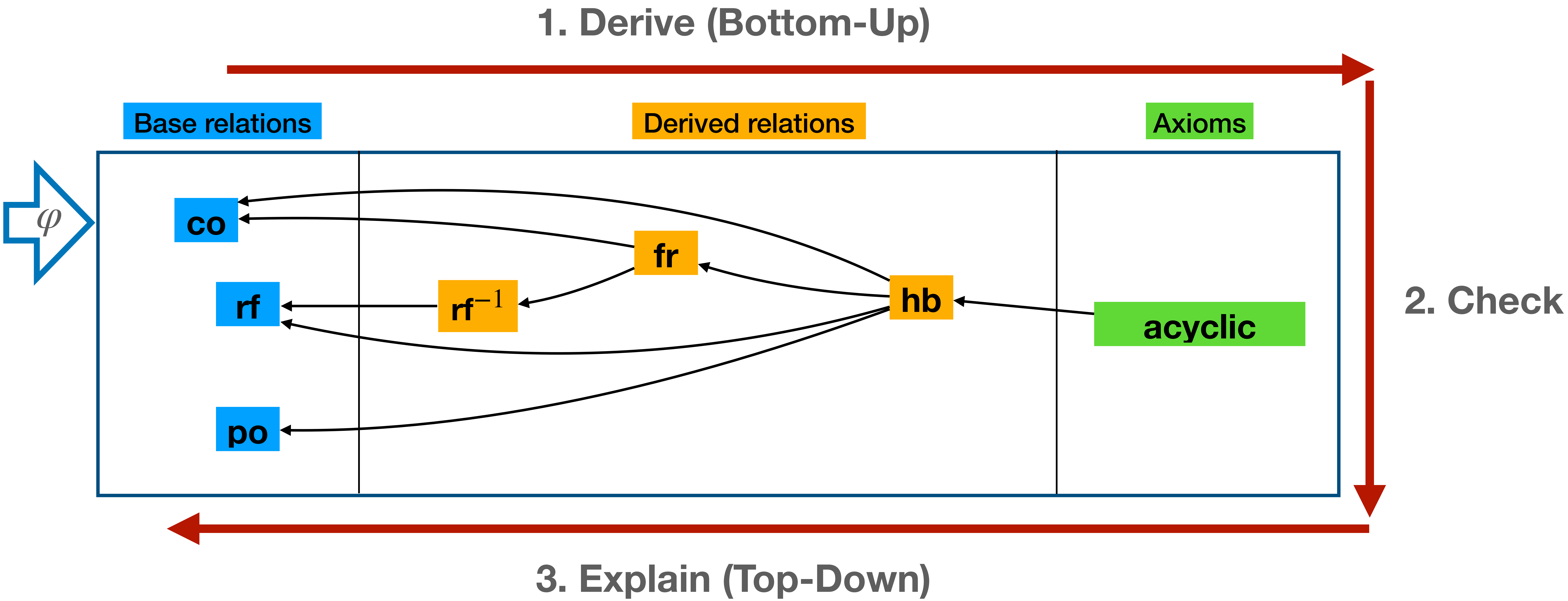

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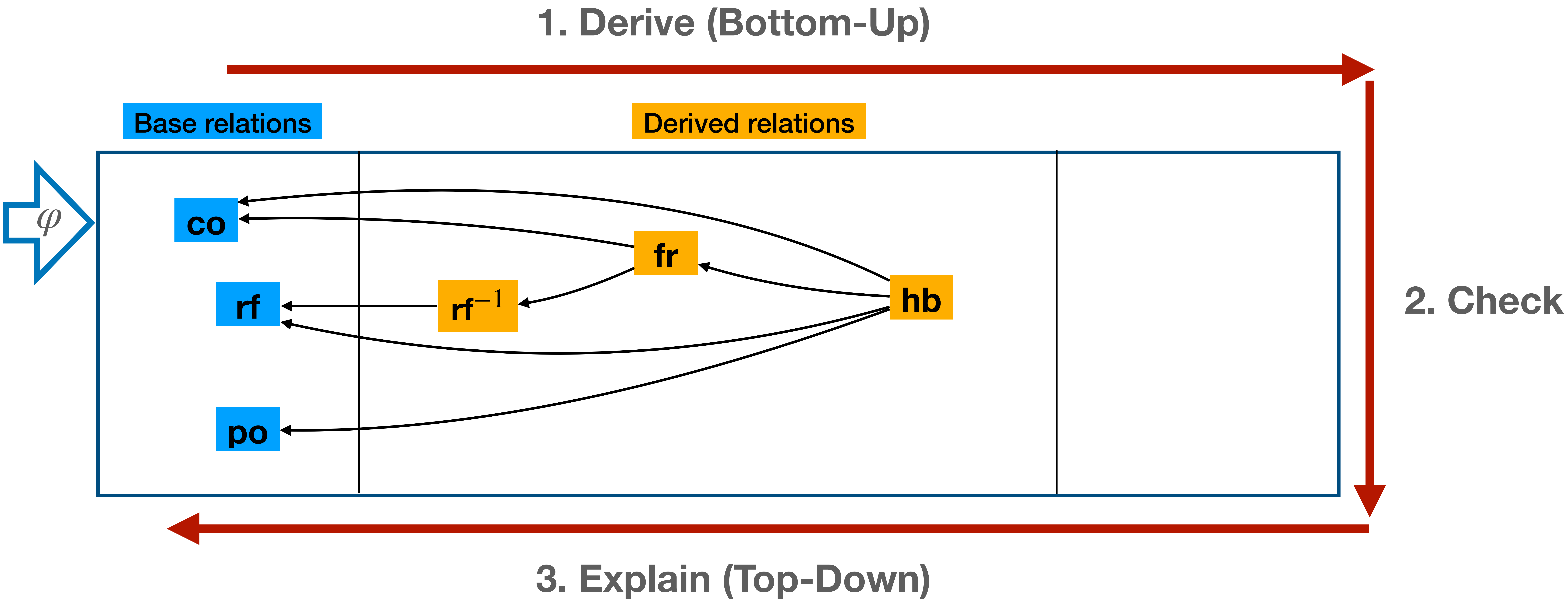

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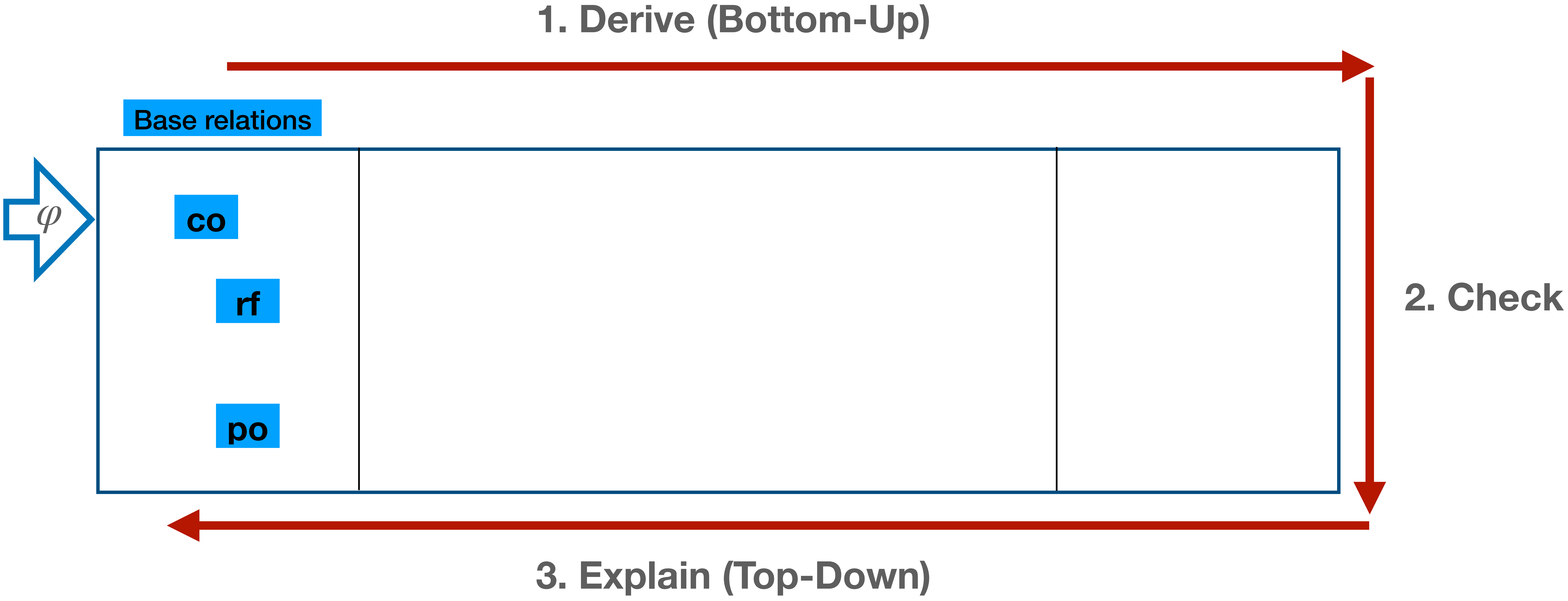

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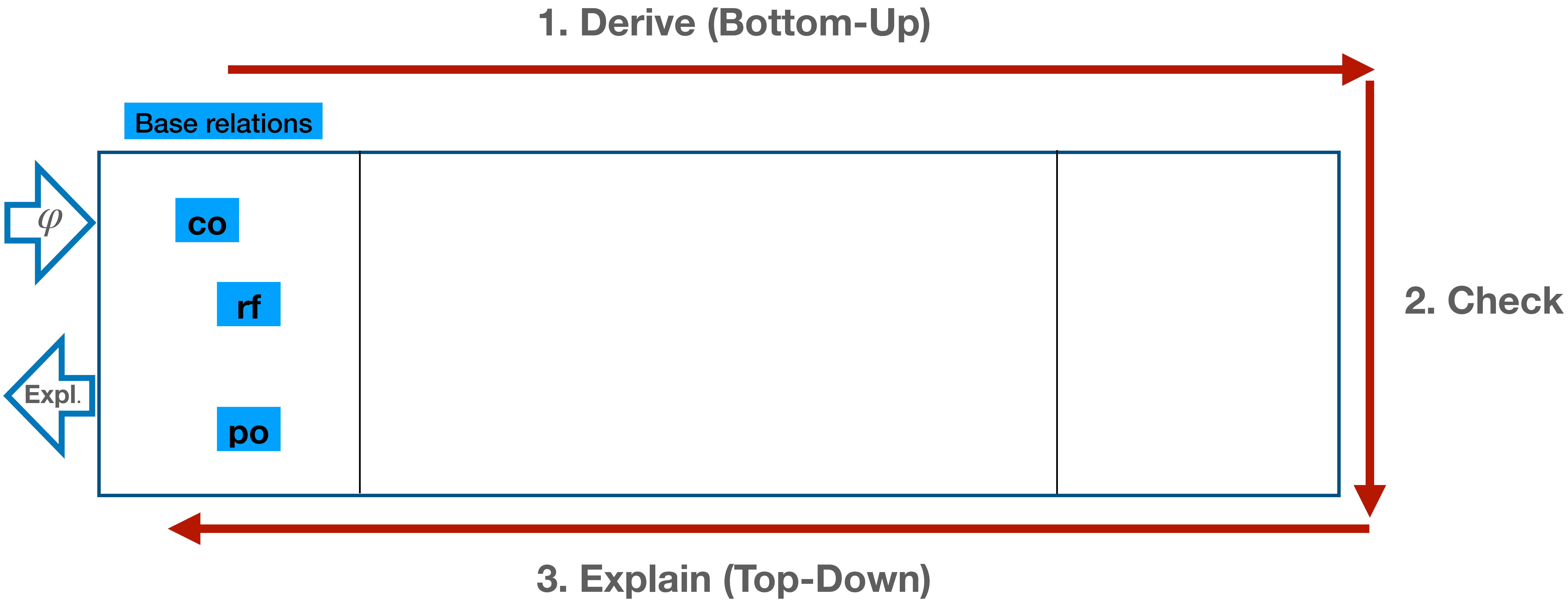

SC

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let fr = rf^-1;co
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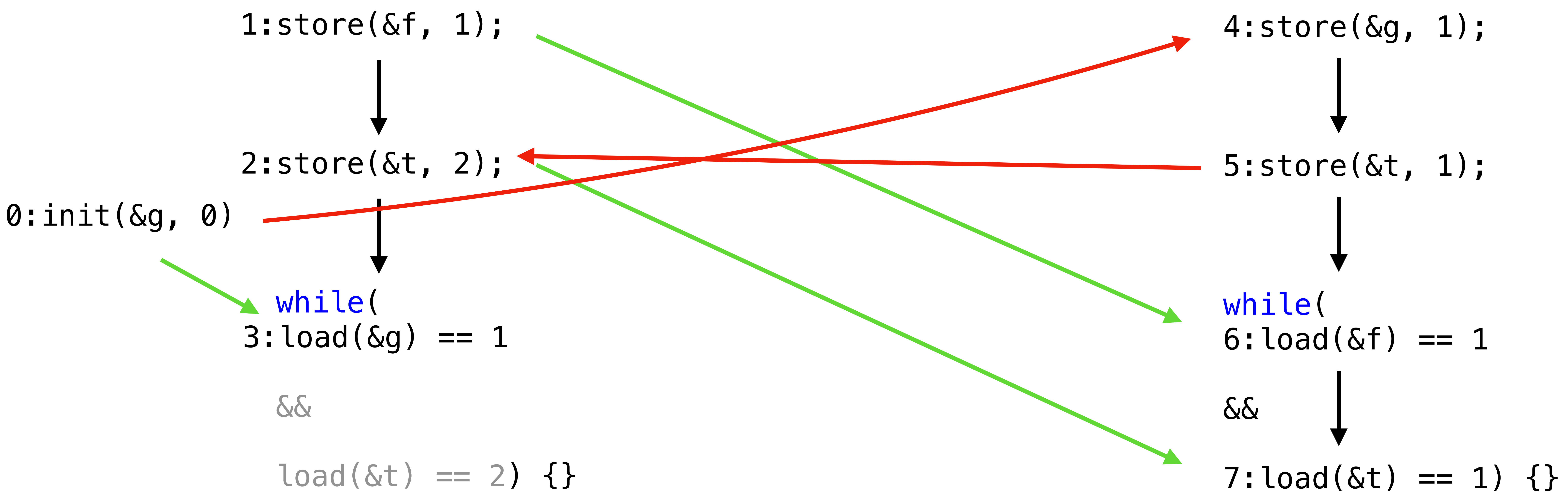
acyclic hb
```

Program order:

Read-from:

Coherence:



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Program order:
Read-from:
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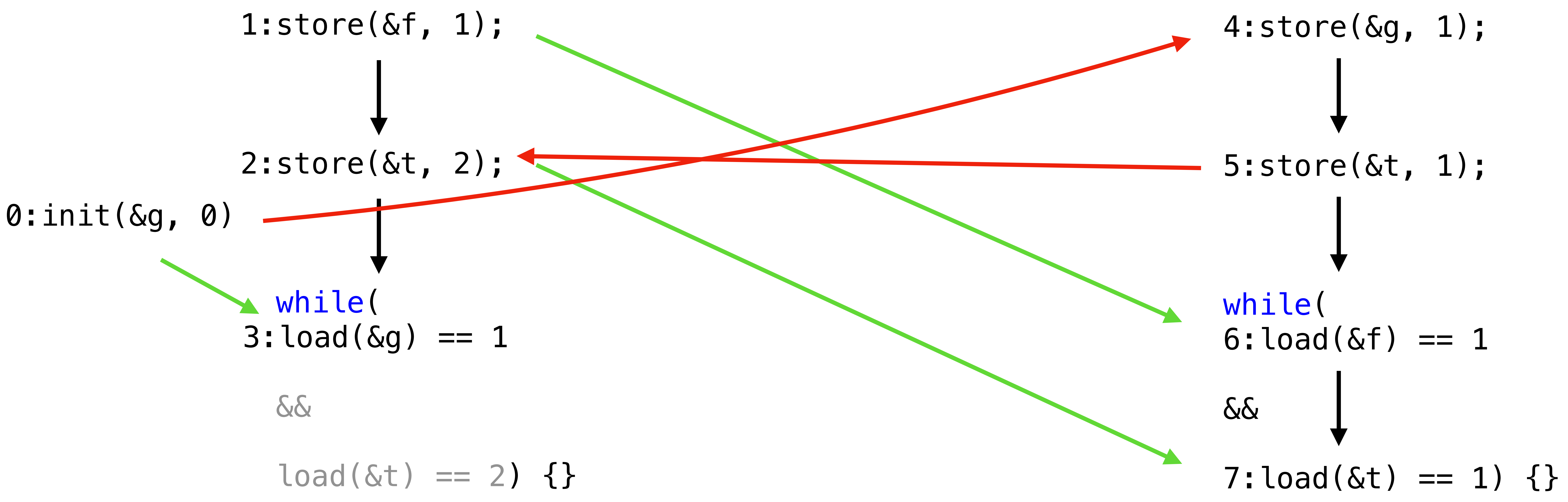
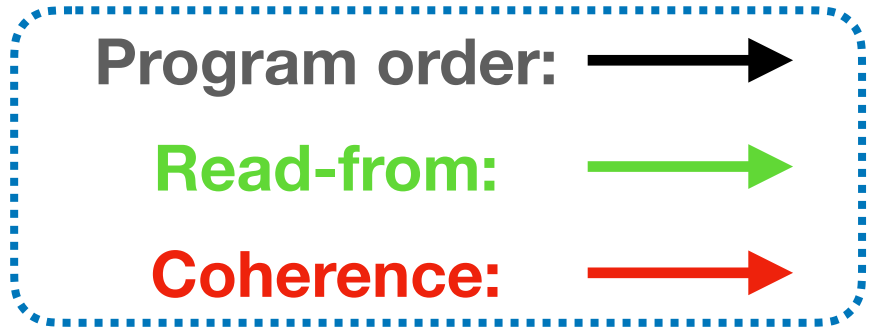
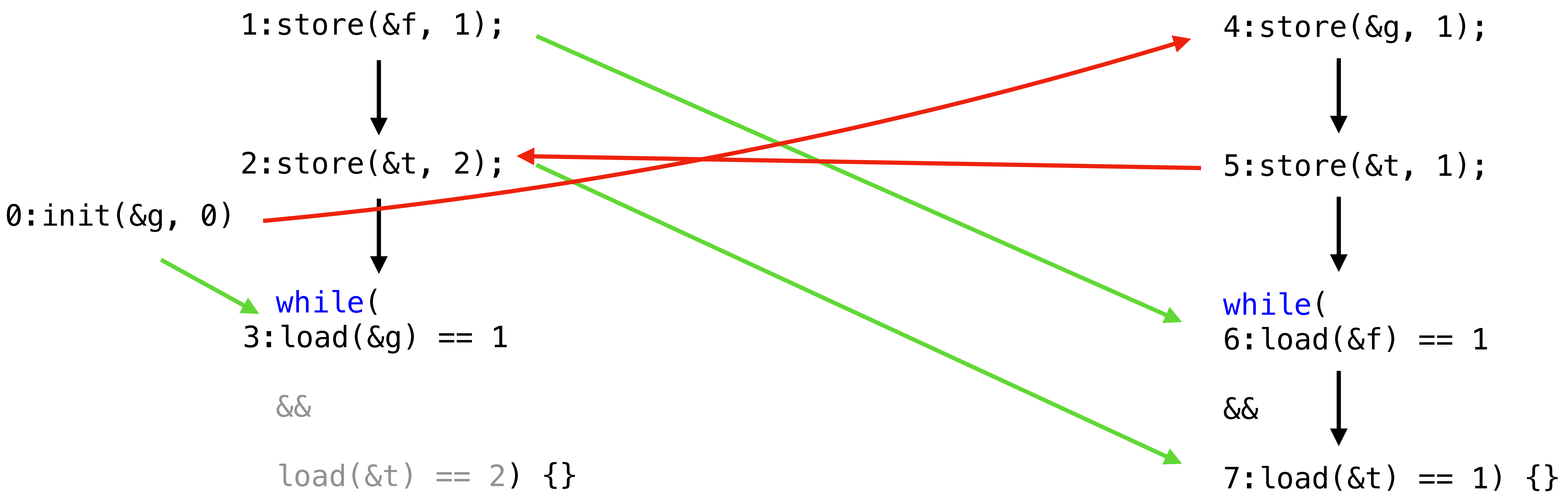


$$\varphi = \begin{array}{c} po(1,2) \wedge po(2,3) \wedge po(4,5) \wedge po(5,6) \wedge po(6,7) \\ \wedge\, rf(0,3) \wedge rf(1,6) \wedge rf(2,7) \\ \wedge\, co(0,4) \wedge co(5,2) \end{array}$$

1. Derive (Bottom-Up)



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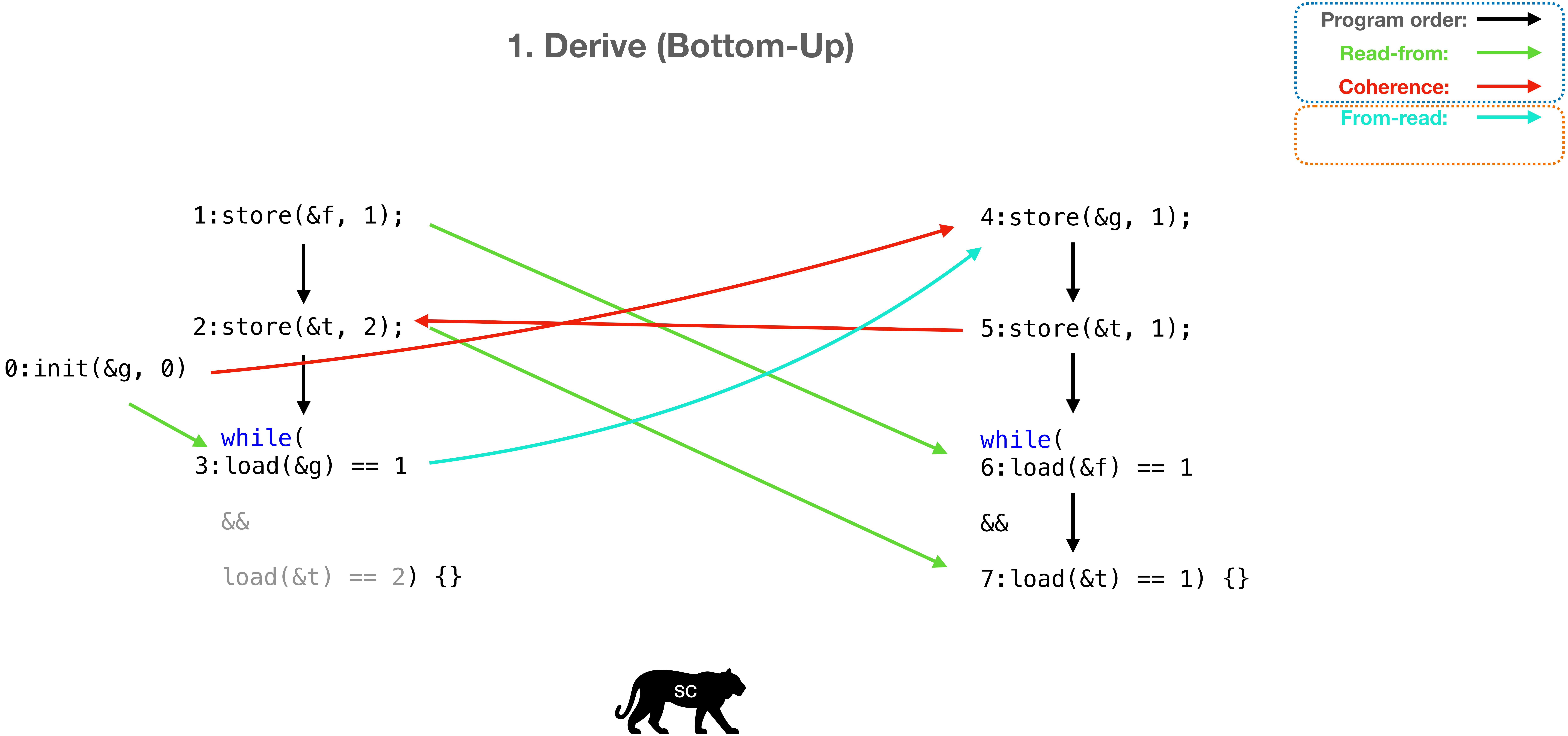


```
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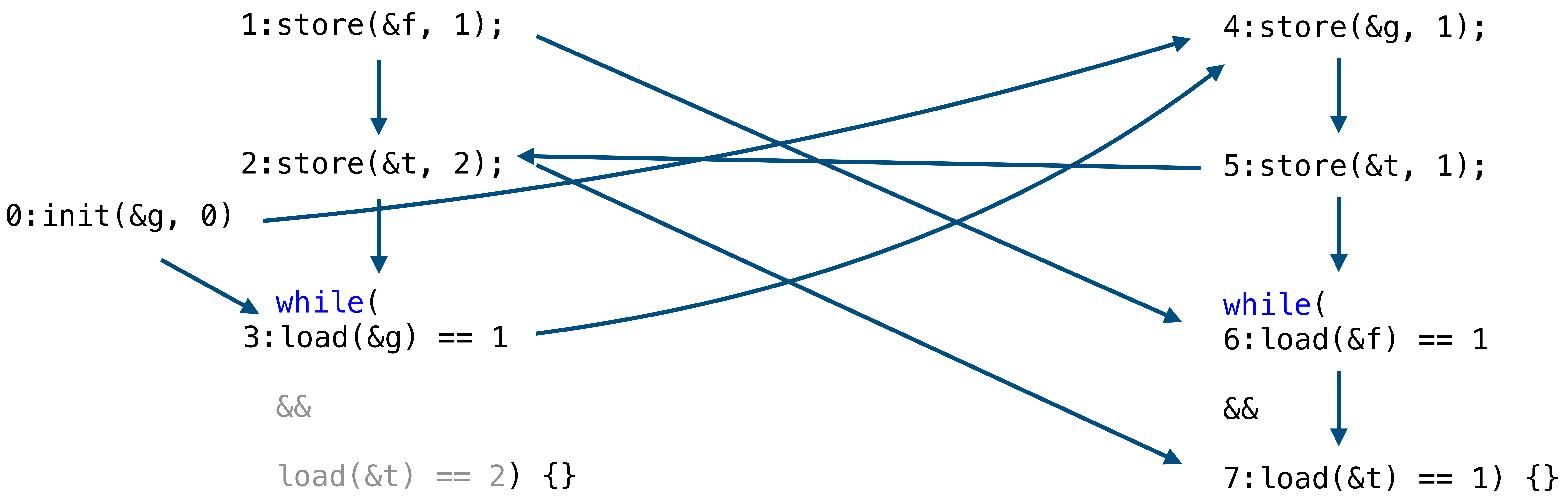
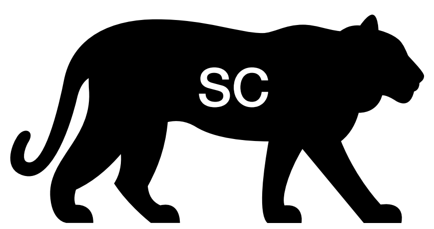
acyclic hb
```

1. Derive (Bottom-Up)

Program order:
Read-from:
Coherence:
From-read:
Happens-before:



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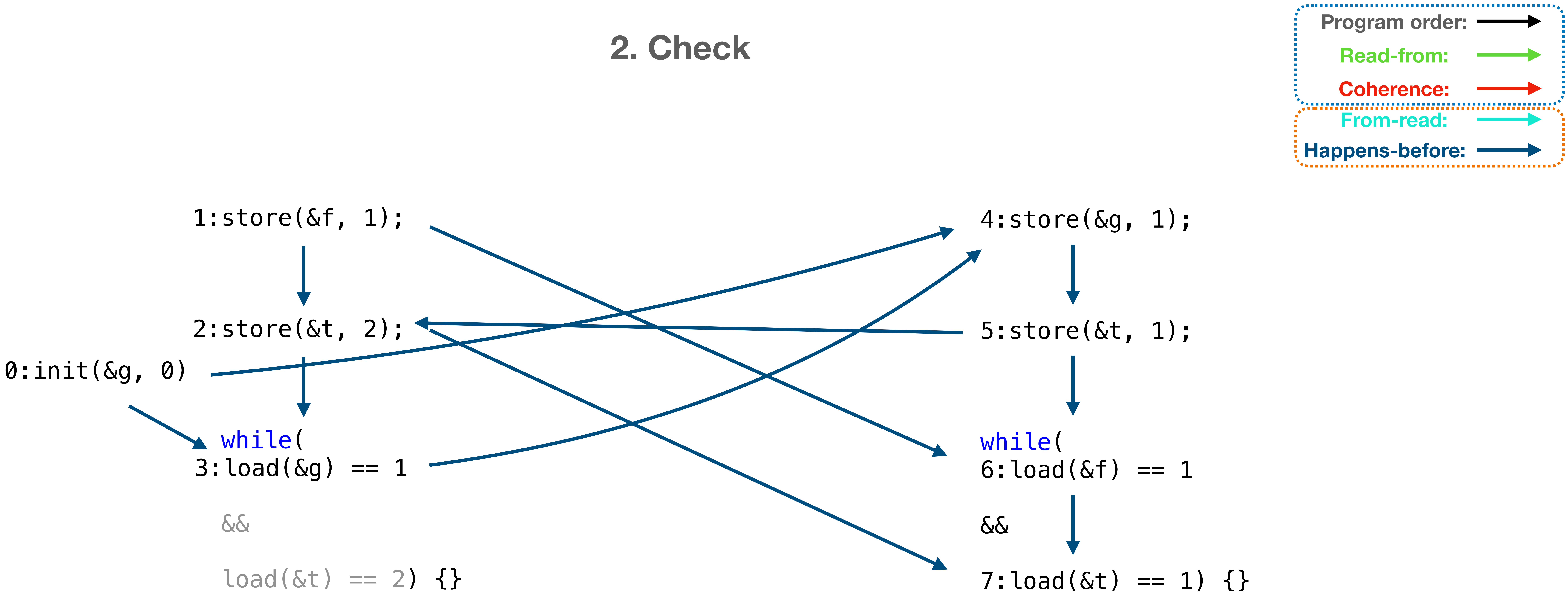
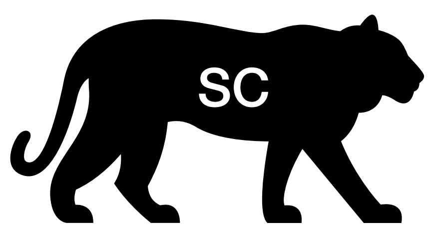
```

2. Check



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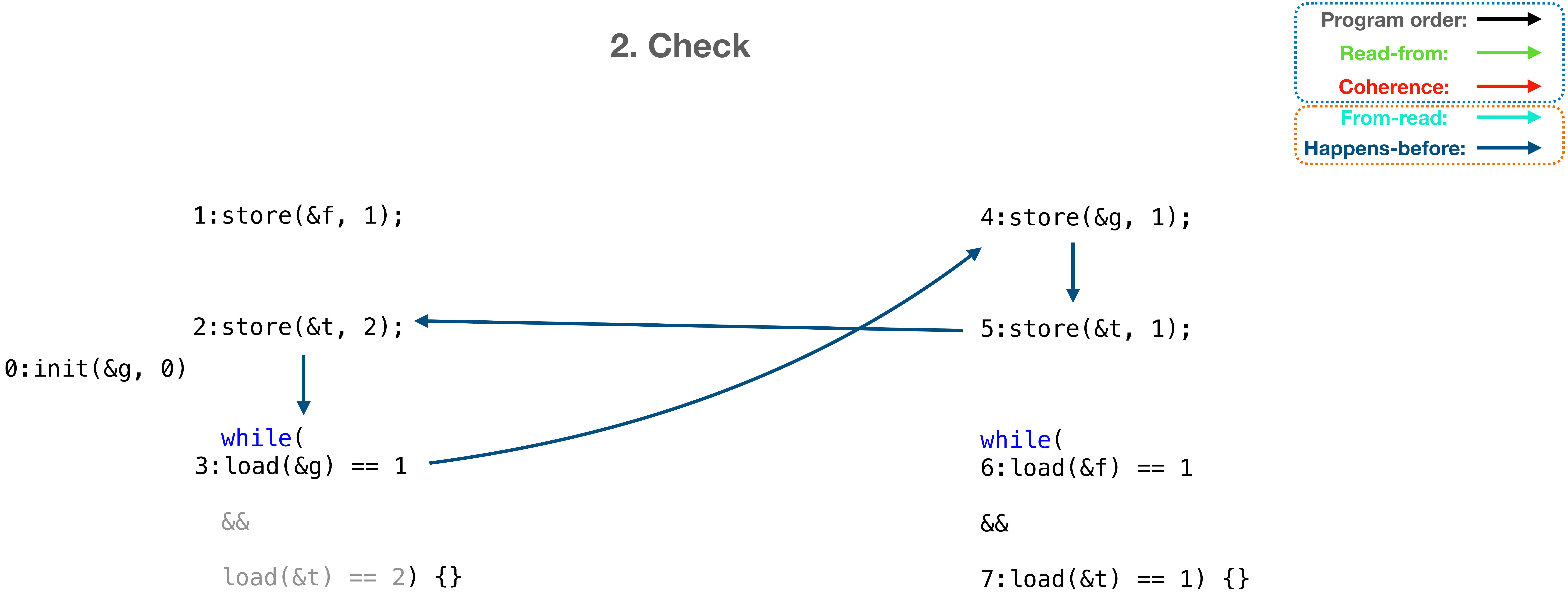
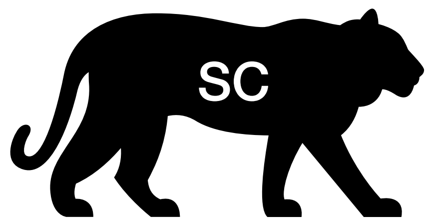





```
let fr = rf^-1;co
let hb = po | rf | fr | co
acyclic hb
```

3. Explain (Top-Down)

Program order:
Read-from:
Coherence:
From-read:
Happens-before:

```
1:store(&f, 1);

2:store(&t, 2);

0:init(&g, 0)

  while(
3:load(&g) == 1

  &&

  load(&t) == 2) {}
```

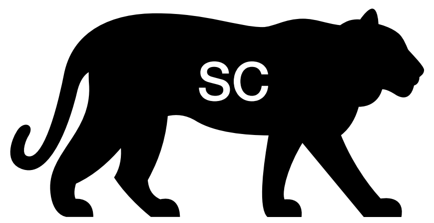
```
4:store(&g, 1);

5:store(&t, 1);

while(
6:load(&f) == 1

&&

7:load(&t) == 1) {}
```

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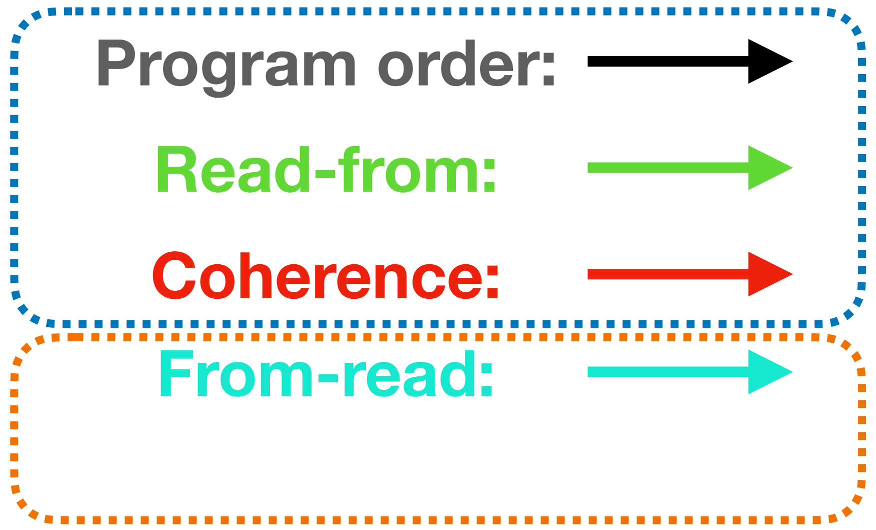

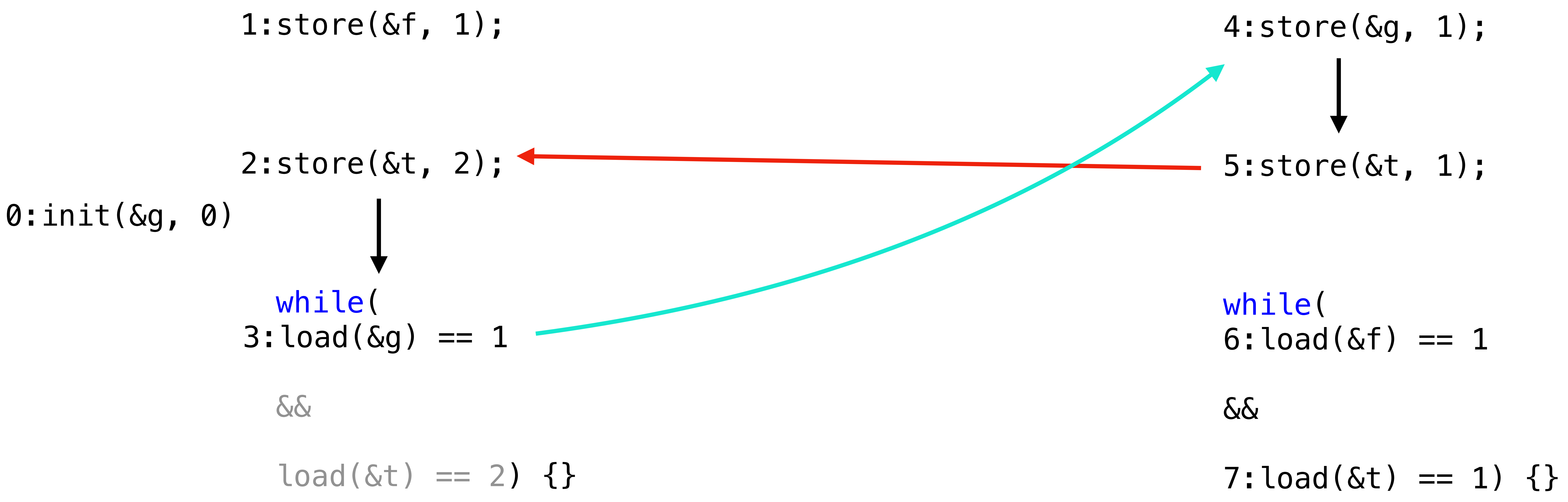

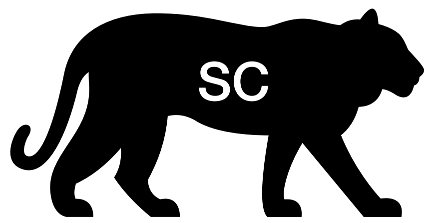


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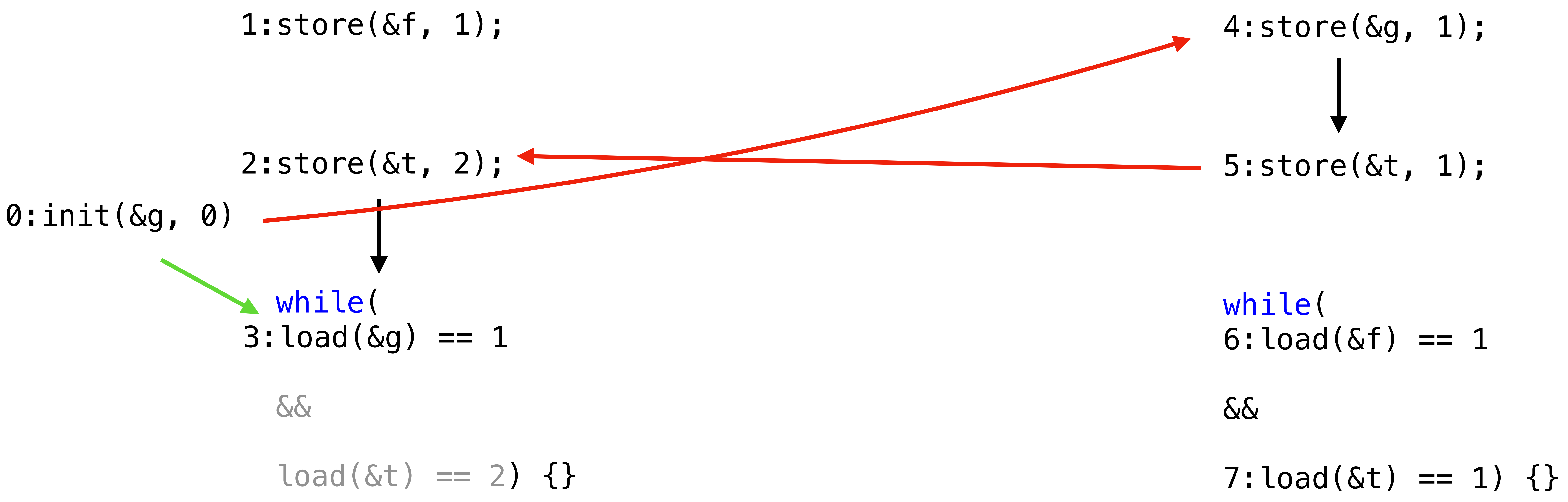
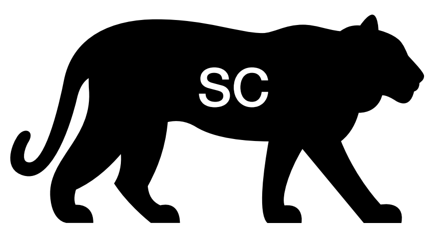
Program order: →
Read-from: →
Coherence: →



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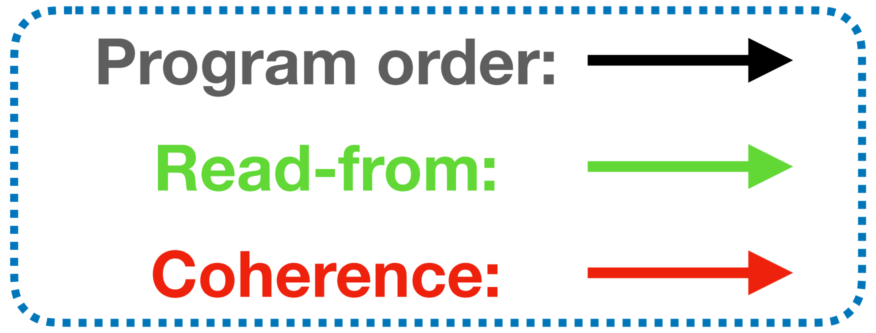

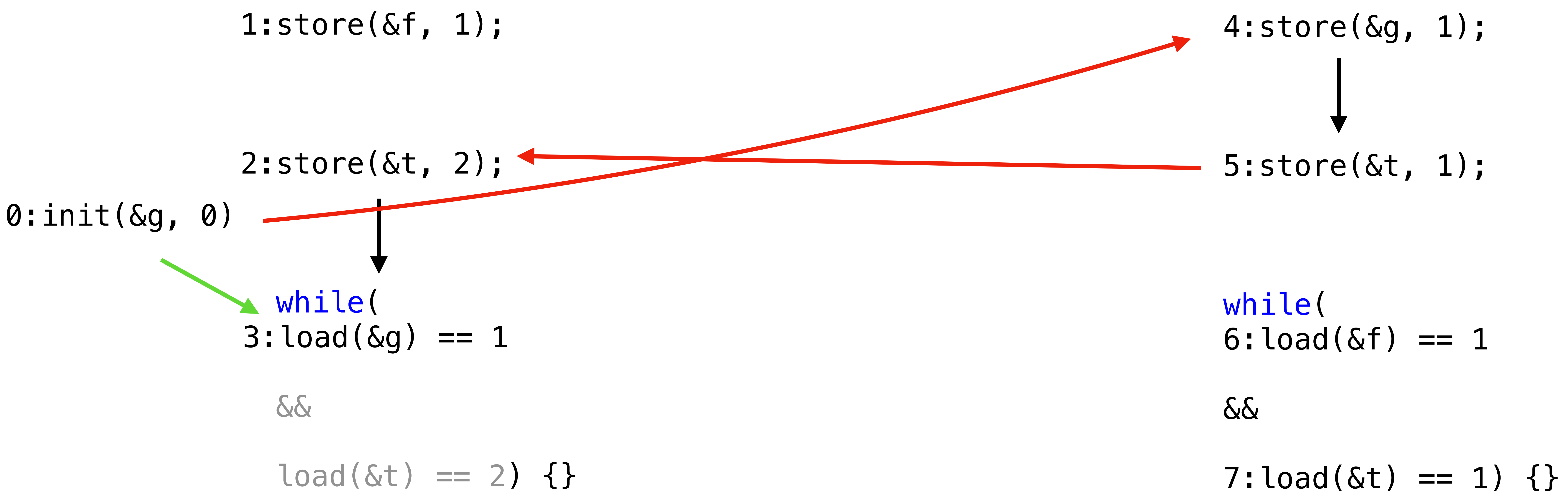

$$\left[\begin{array}{c} po(2,3) \wedge po(4,5) \\ \wedge\, rf(0,3) \\ \wedge\, co(0,4) \wedge co(5,2) \end{array}\right] \sqsubseteq \left[\begin{array}{c} po(1,2) \wedge po(2,3) \wedge po(4,5) \wedge po(5,6) \wedge po(6,7) \\ \wedge\, rf(0,3) \wedge rf(1,6) \wedge rf(2,7) \\ \wedge\, co(0,4) \wedge co(5,2) \end{array}\right] = \varphi$$

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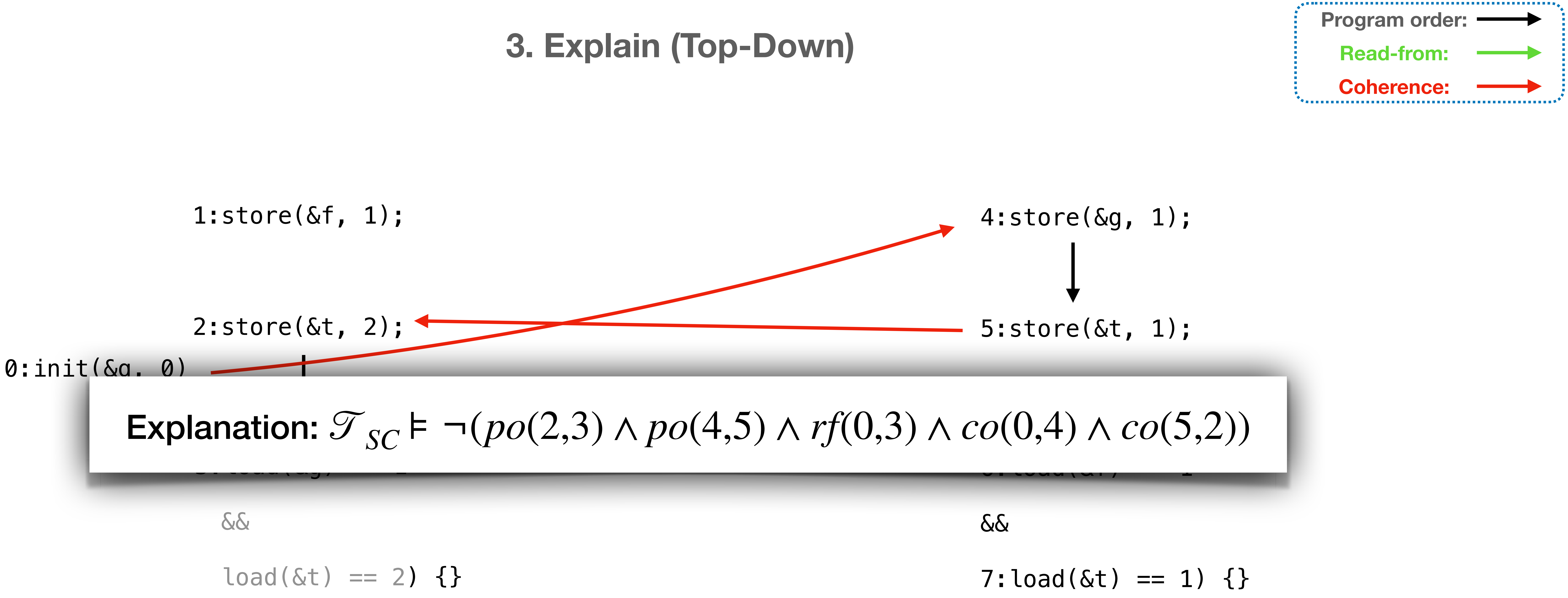

Explanation: $\mathcal{T}_{SC} \vDash \neg(po(2,3) \wedge po(4,5) \wedge rf(0,3) \wedge co(0,4) \wedge co(5,2))$

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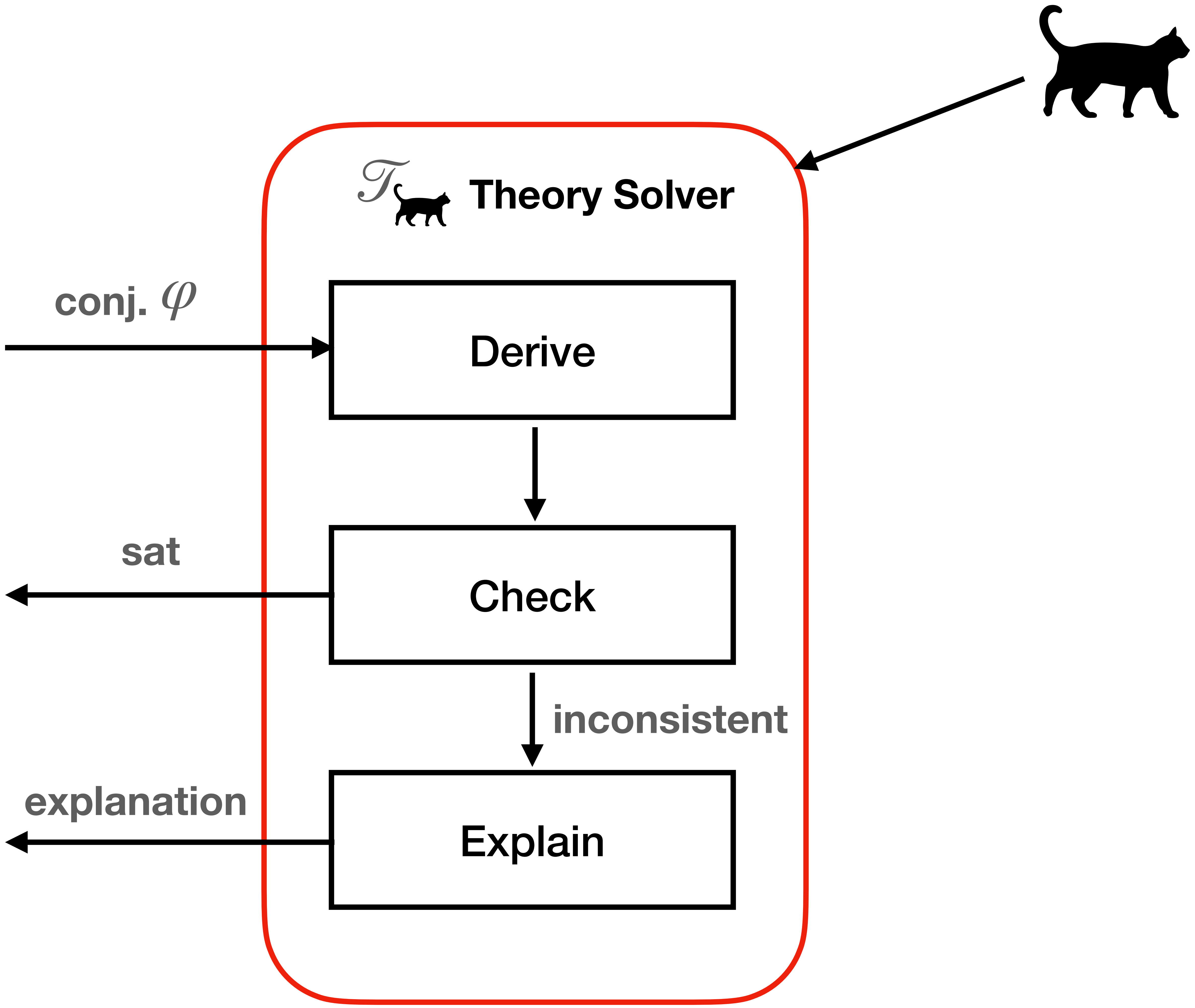
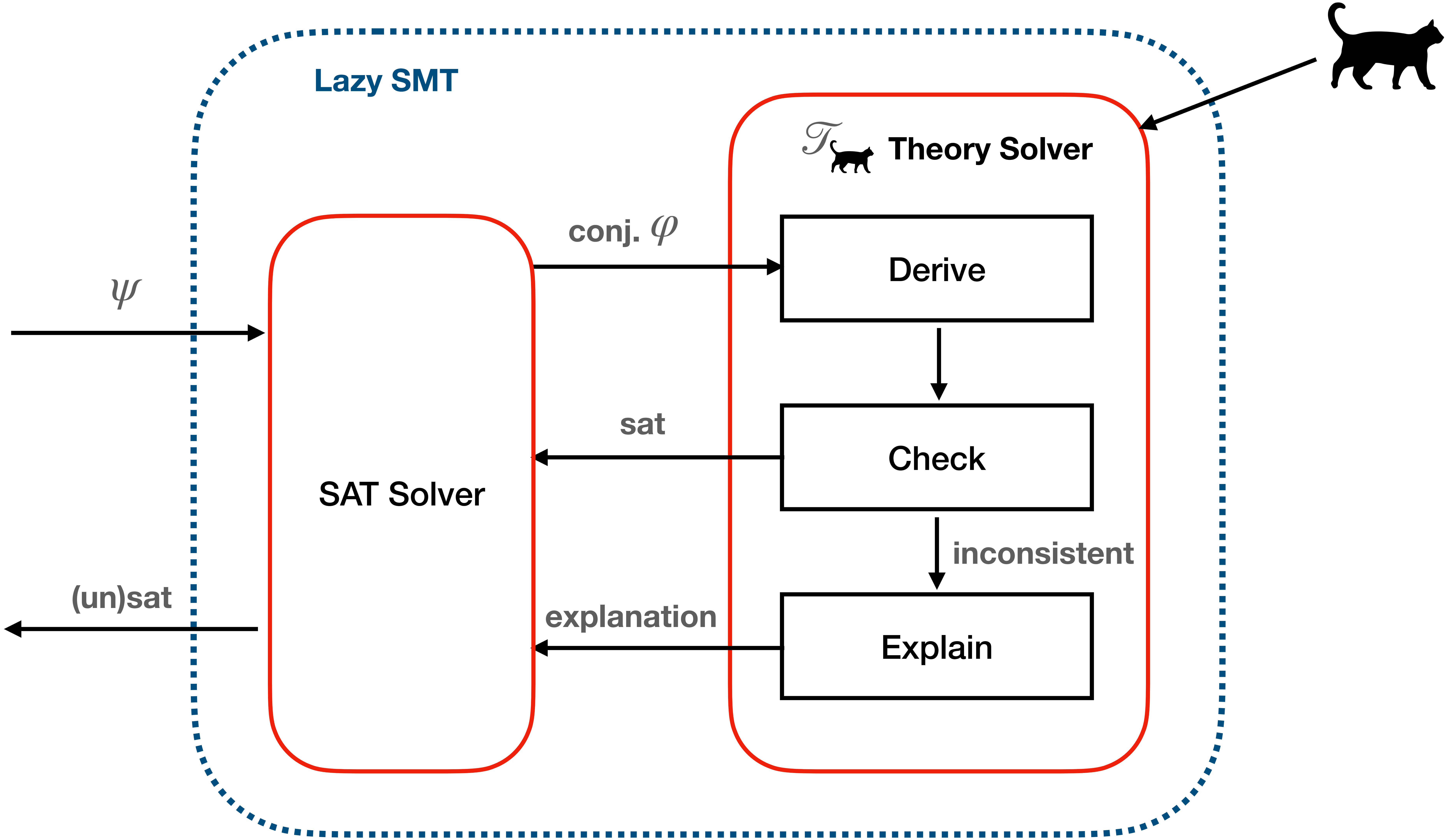
A Theory Solver for Consistency



A Decision Procedure for Consistency



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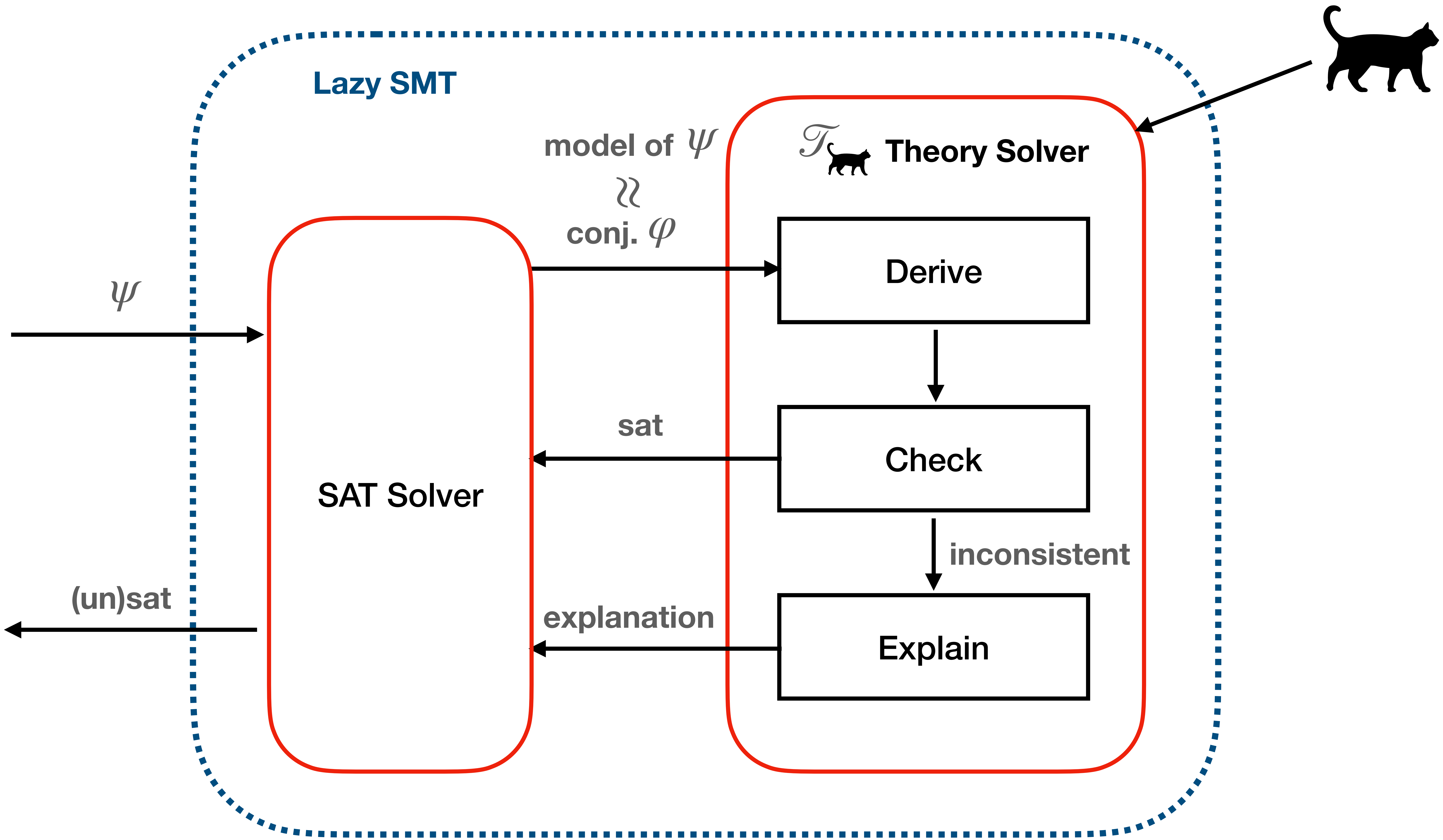

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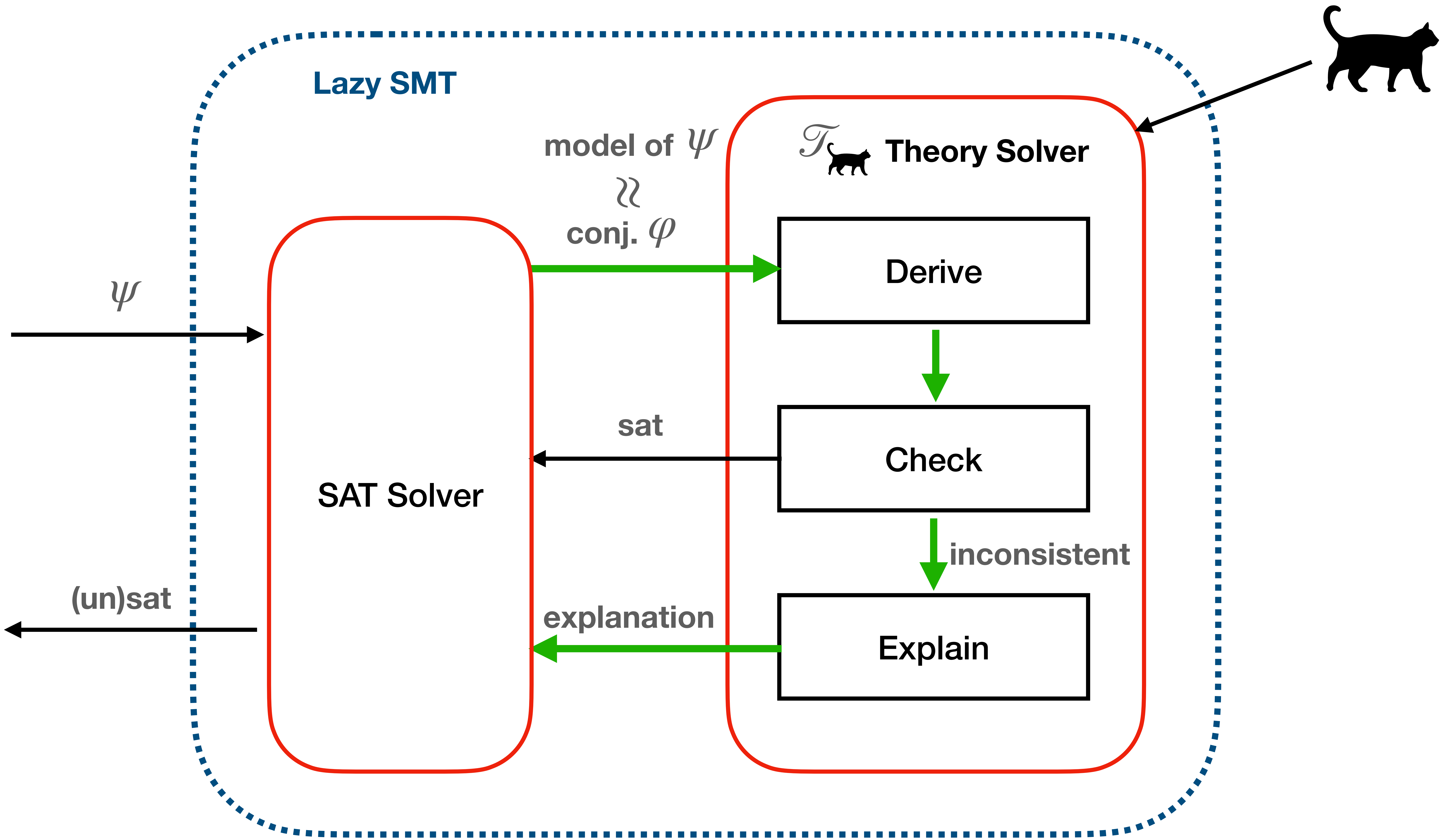

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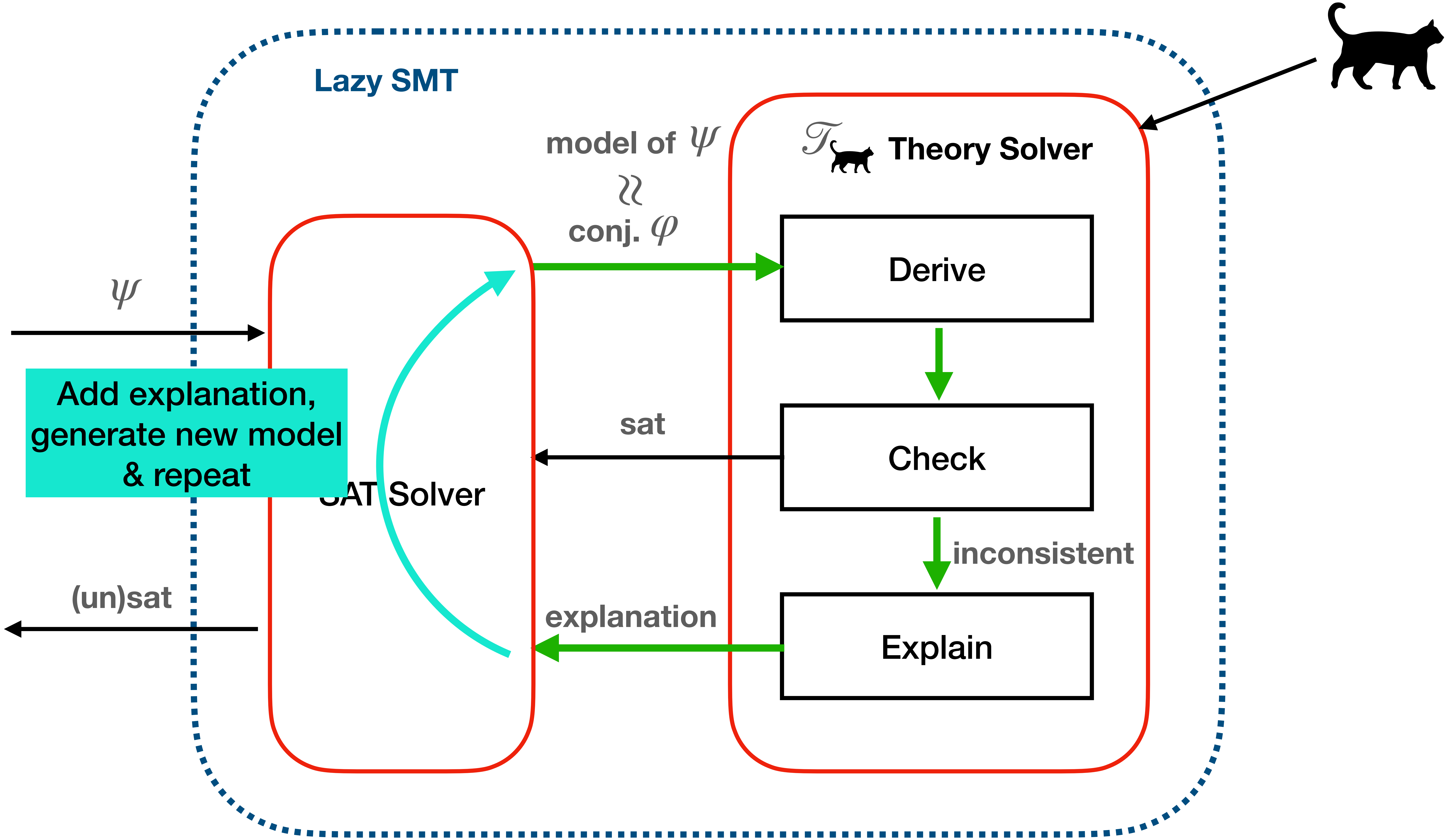
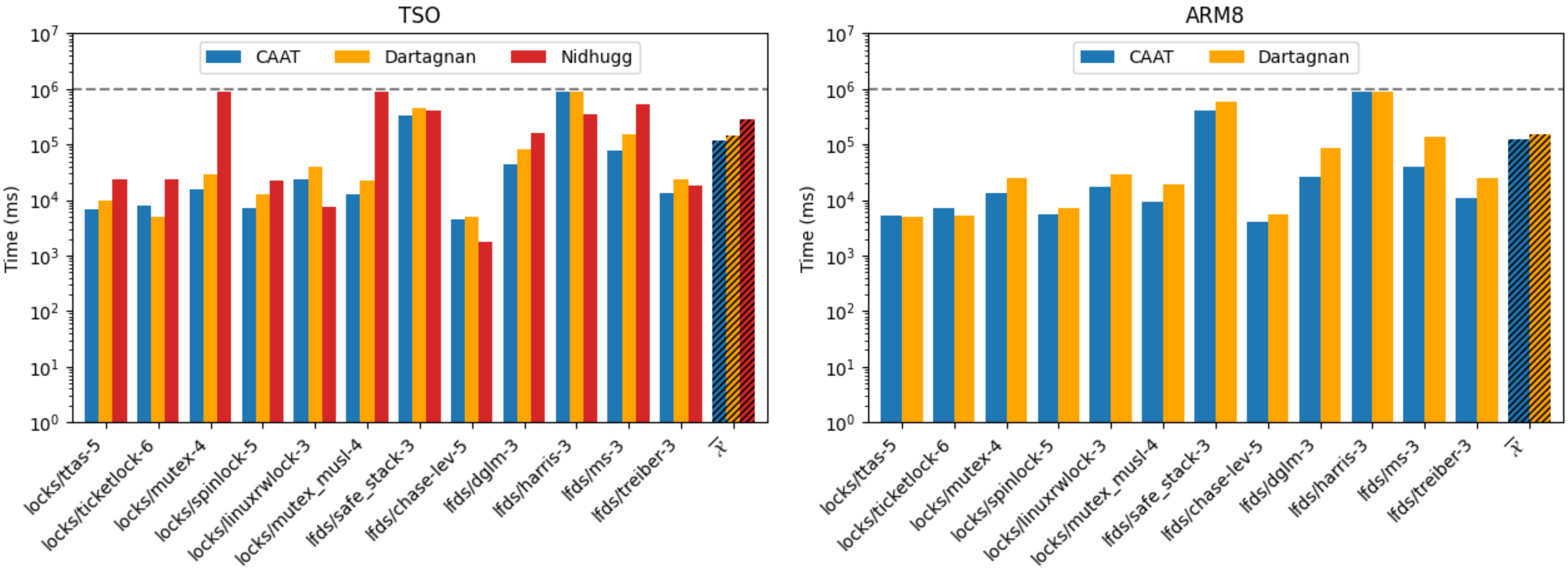


Evaluation

Evaluation: simple CATs 🐈‍⬛

No recursion



Evaluation: simple CATs

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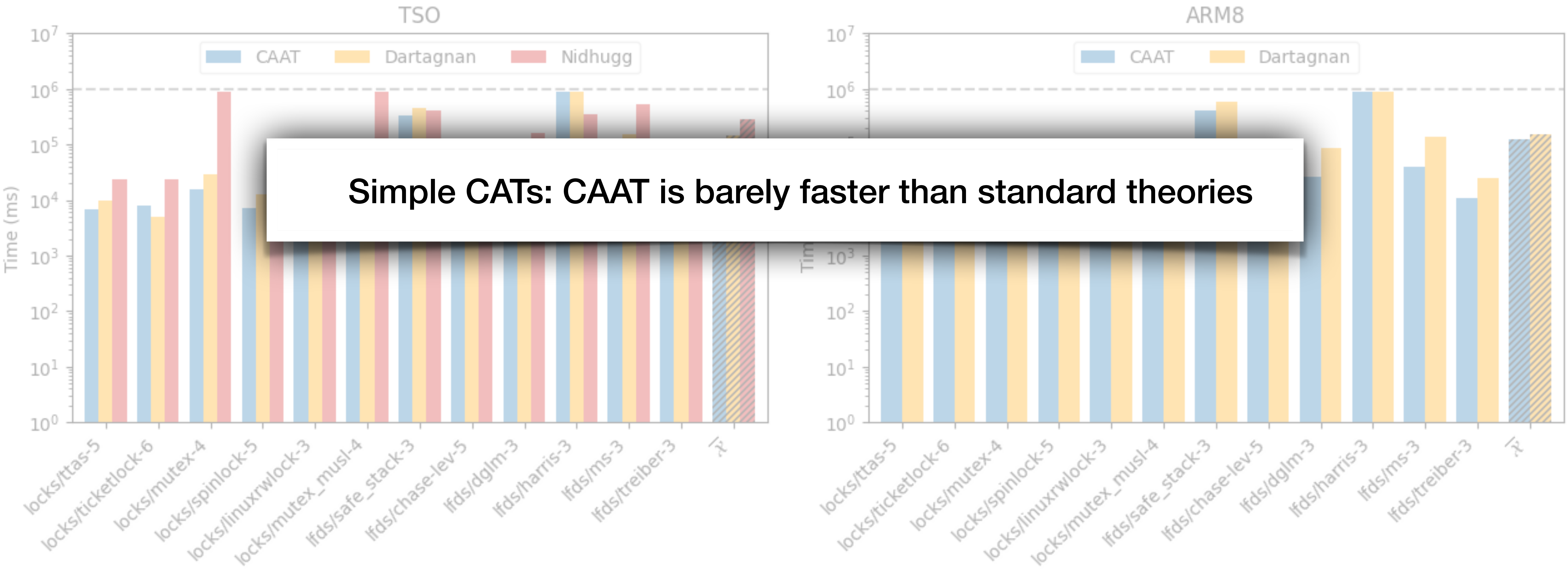
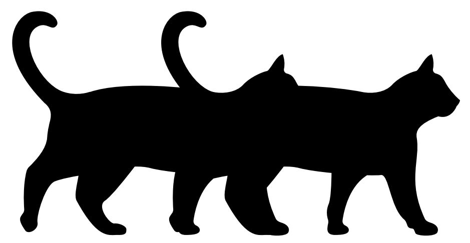
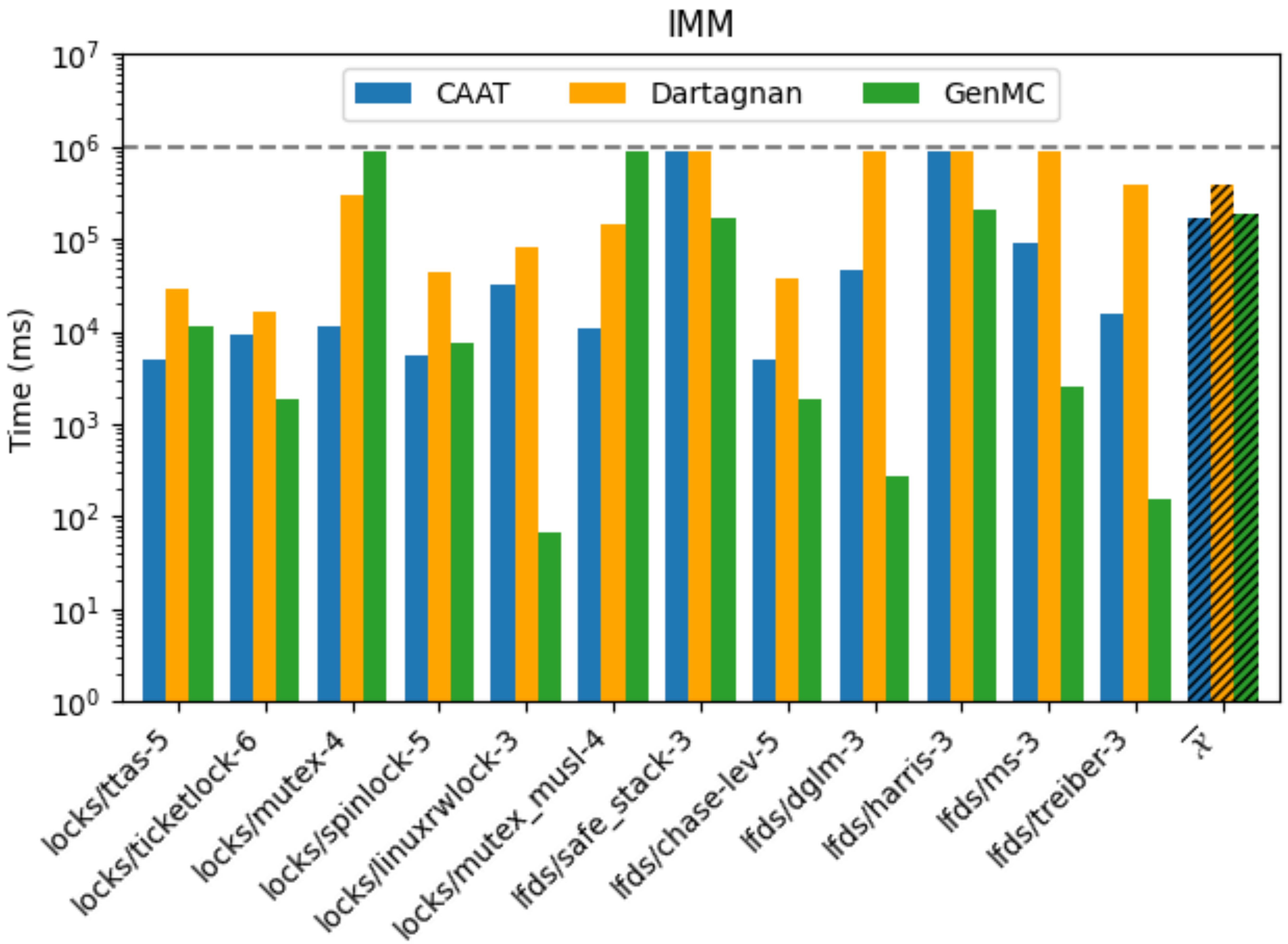
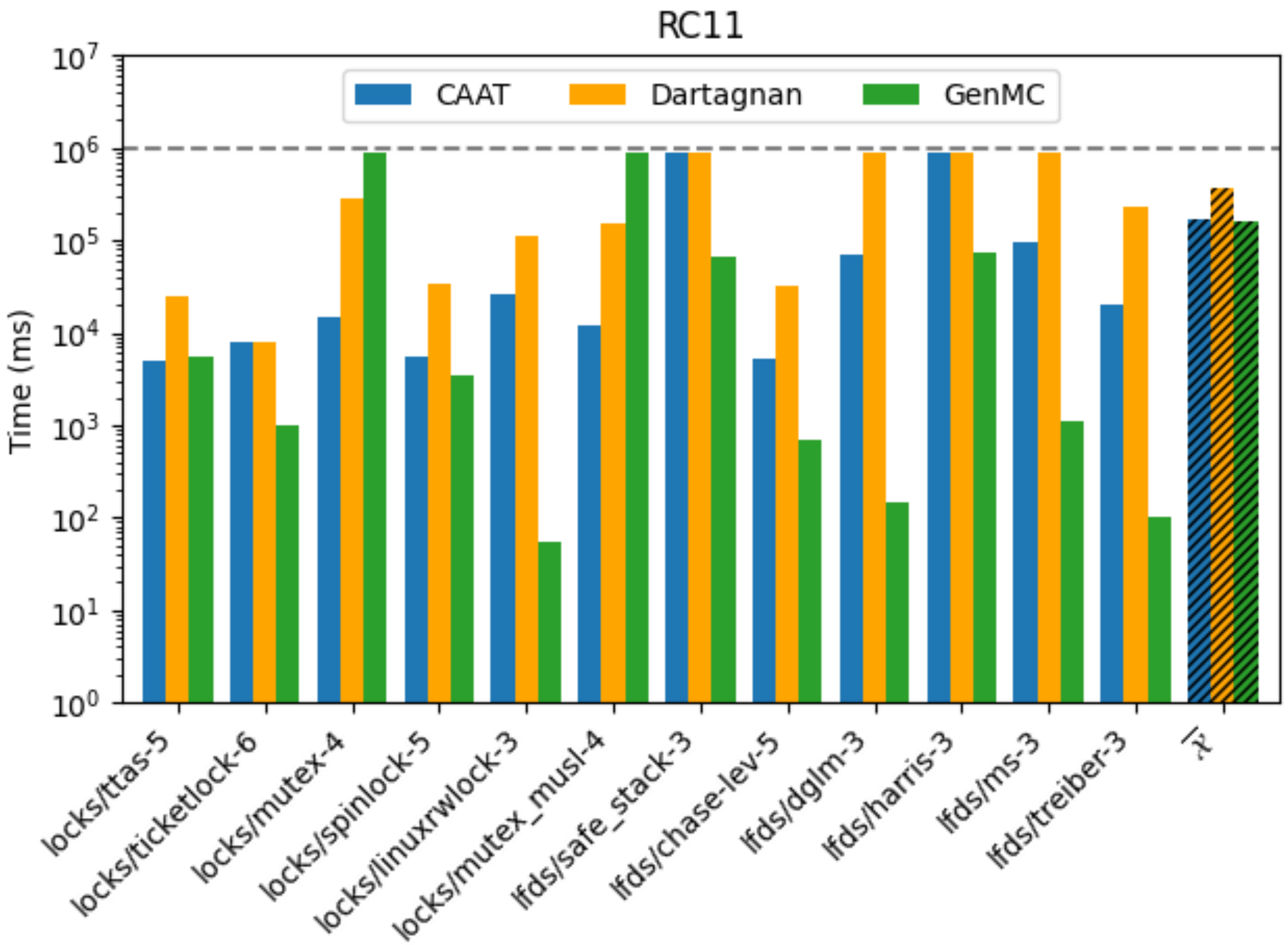


Simple CATs: CAAT is barely faster than standard theories

Evaluation: complex CATs 

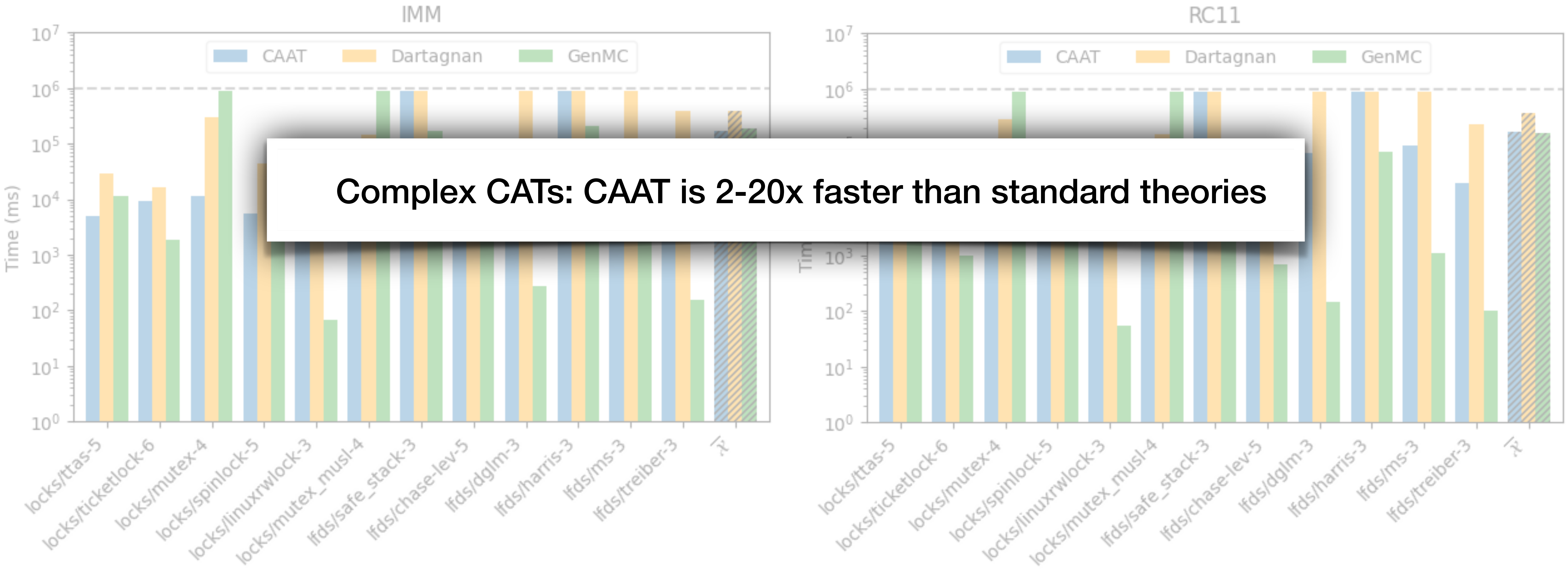
Linear recursion





Evaluation: complex CATs

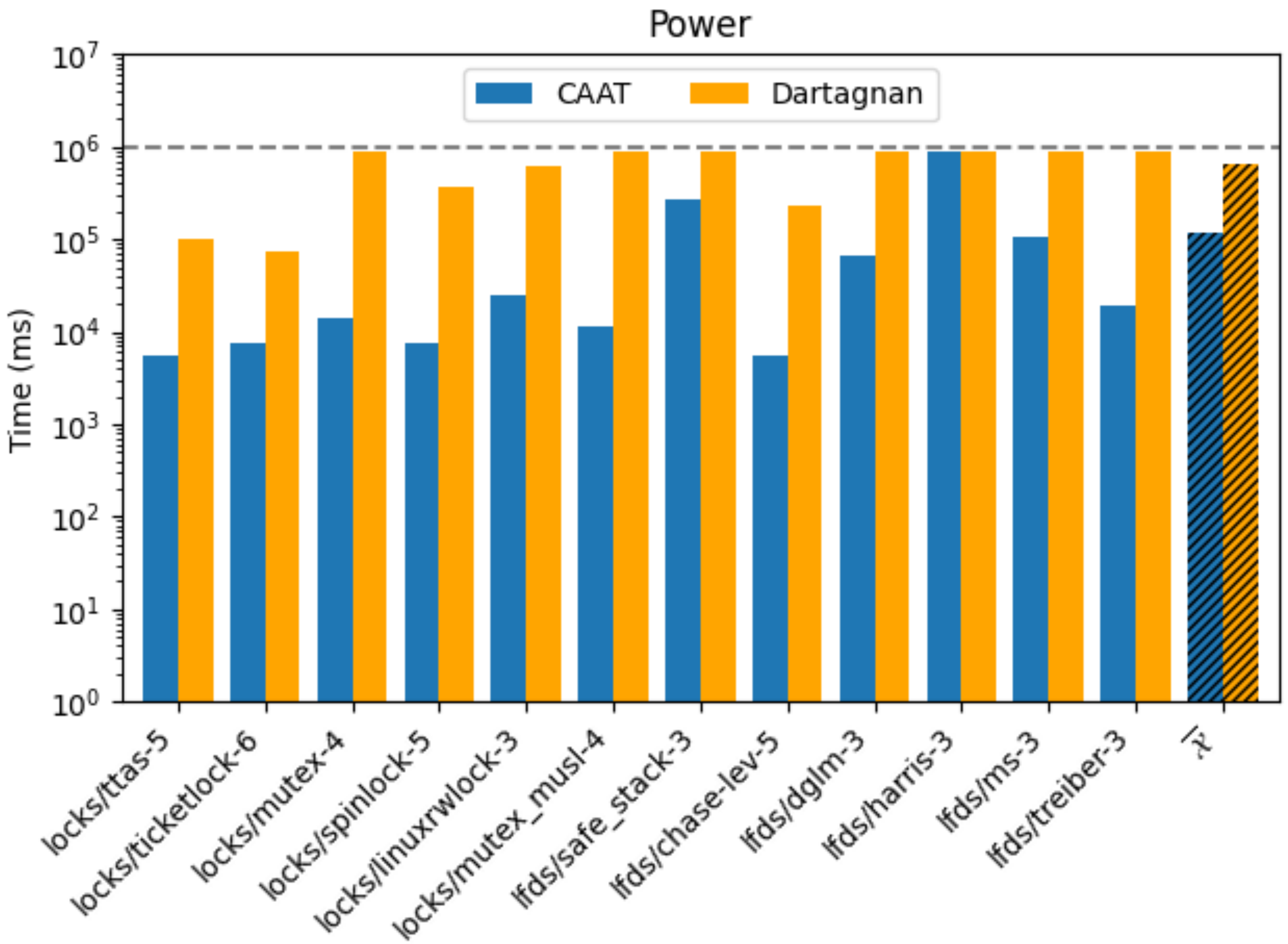
Linear recursion



Complex CATs: CAAT is 2-20x faster than standard theories

Evaluation: very complex CATs

Non-linear recursion



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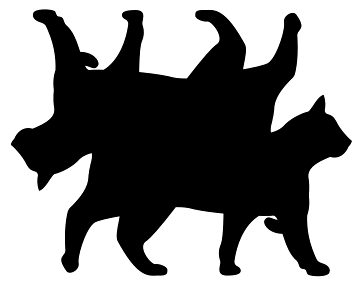

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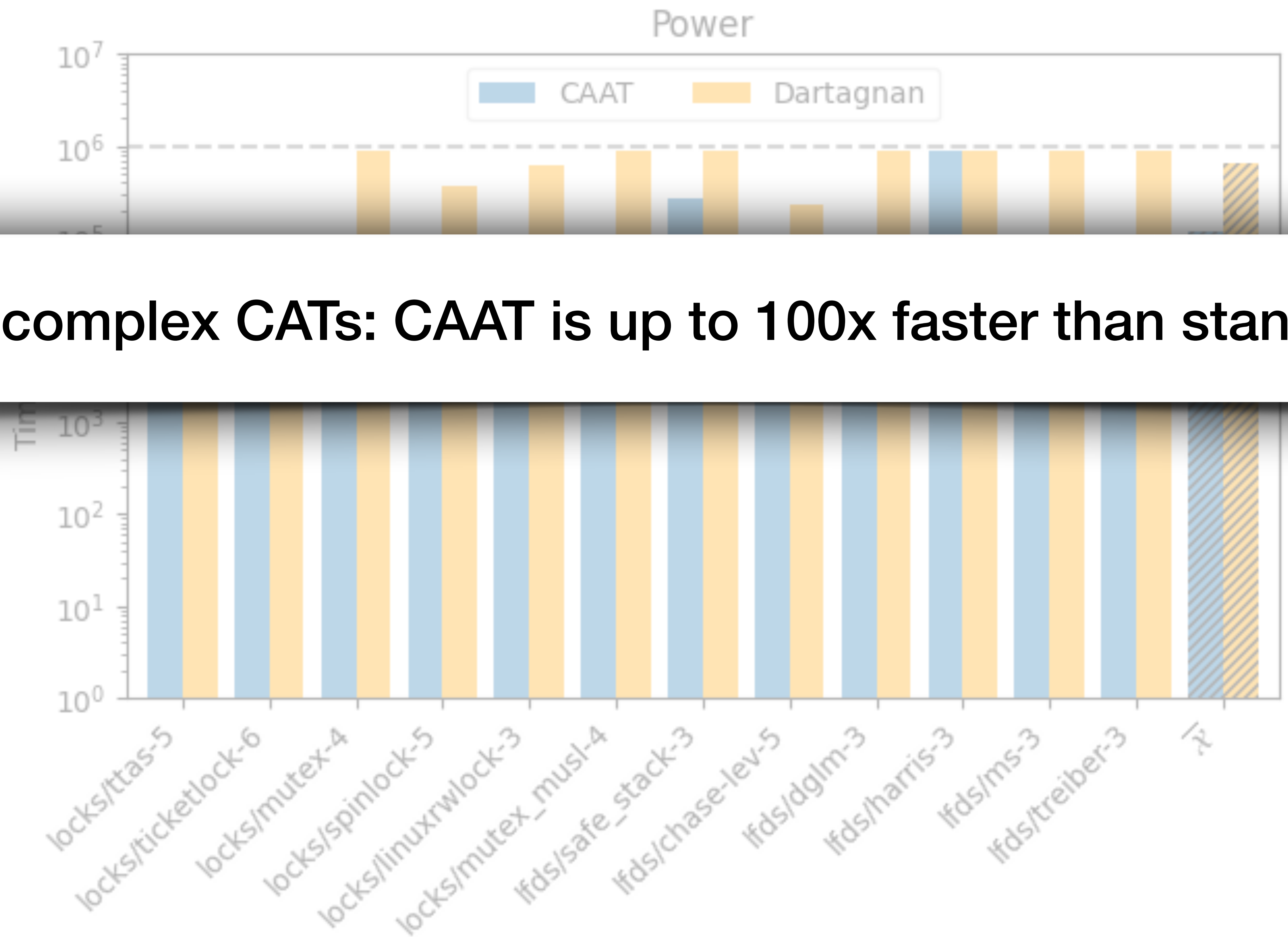
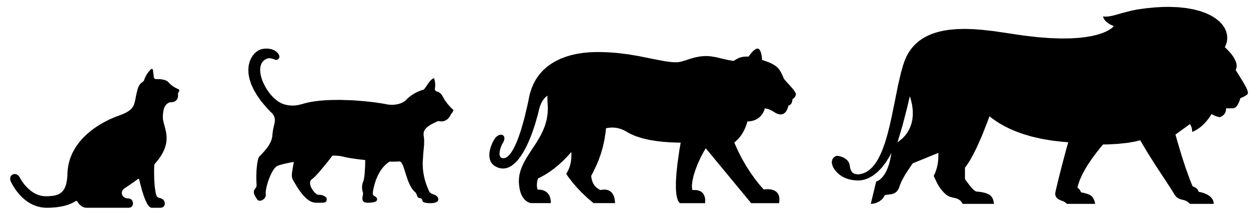


Very complex CATs: CAAT is up to 100x faster than standard theories

Conclusion

- We see consistency models as a family of theories 
- Consistency theories handle least fixed points, unlike existing theories
- We give a general theory solver for consistency theories
- Using CAAT in BMC gives substantial performance improvement

Ongoing Work

- Online integration with the SMT solver
- Incrementality is a problem — the partial models are often largely different, because other theories make the solver backtrack!
- Use matching instead!

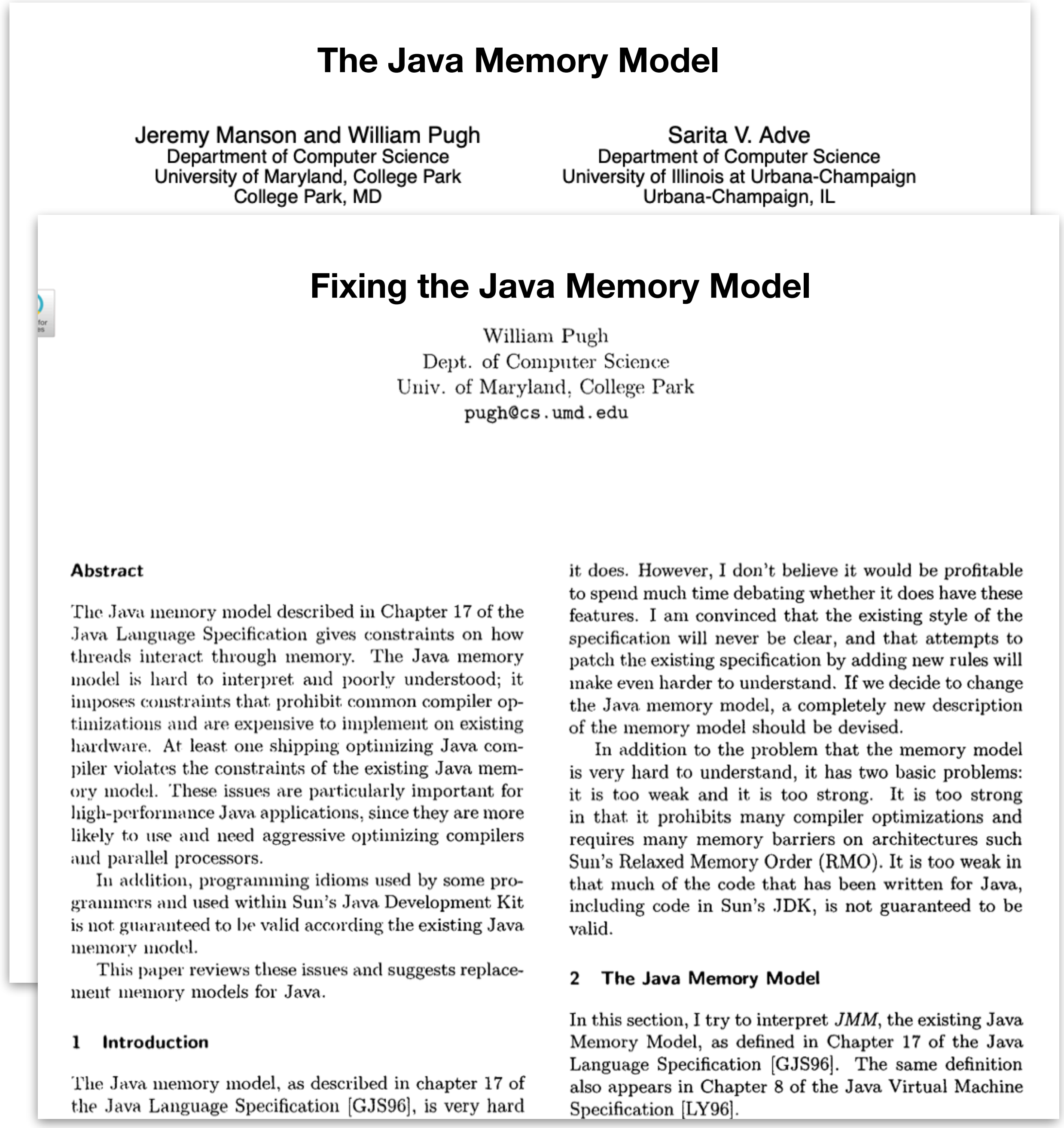
Cyclic Proofs for Axiomatic Memory Models

ongoing work with Jan Grünke and Thomas Haas

Memory Models have Bugs

Memory Models have Bugs

- Java MM *[JLS 1996]*
  - too weak to build new synchronization primitives *[Pugh]*
  - too strong for common compiler optimizations (i.e. CSE) *[Pugh]*



The Java Memory Model

Jeremy Manson and William Pugh
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Fixing the Java Memory Model

William Pugh
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Abstract

The Java memory model described in Chapter 17 of the Java Language Specification gives constraints on how threads interact through memory. The Java memory model is hard to interpret and poorly understood; it imposes constraints that prohibit common compiler optimizations and are expensive to implement on existing hardware. At least one shipping optimizing Java compiler violates the constraints of the existing Java memory model. These issues are particularly important for high-performance Java applications, since they are more likely to use and need aggressive optimizing compilers and parallel processors.

In addition, programming idioms used by some programmers and used within Sun's Java Development Kit is not guaranteed to be valid according the existing Java memory model.

This paper reviews these issues and suggests replacement memory models for Java.

1 Introduction

The Java memory model, as described in chapter 17 of the Java Language Specification [GJS96], is very hard

it does. However, I don't believe it would be profitable to spend much time debating whether it does have these features. I am convinced that the existing style of the specification will never be clear, and that attempts to patch the existing specification by adding new rules will make even harder to understand. If we decide to change the Java memory model, a completely new description of the memory model should be devised.

In addition to the problem that the memory model is very hard to understand, it has two basic problems: it is too weak and it is too strong. It is too strong in that it prohibits many compiler optimizations and requires many memory barriers on architectures such Sun's Relaxed Memory Order (RMO). It is too weak in that much of the code that has been written for Java, including code in Sun's JDK, is not guaranteed to be valid.

2 The Java Memory Model

In this section, I try to interpret *JMM*, the existing Java Memory Model, as defined in Chapter 17 of the Java Language Specification [GJS96]. The same definition also appears in Chapter 8 of the Java Virtual Machine Specification [LY96].

Memory Models have Bugs

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  - too strong for common compiler optimizations (i.e. CSE) *[Pugh]*
- C/C++11 MM
  - common compiler optimizations are invalid *[Vafeiadis et. al]*
  - allows strange behavior (i.e. OOTA) *[Vafeiadis et. al]*
  - SC fences are too weak *[Sarkar et. al]*
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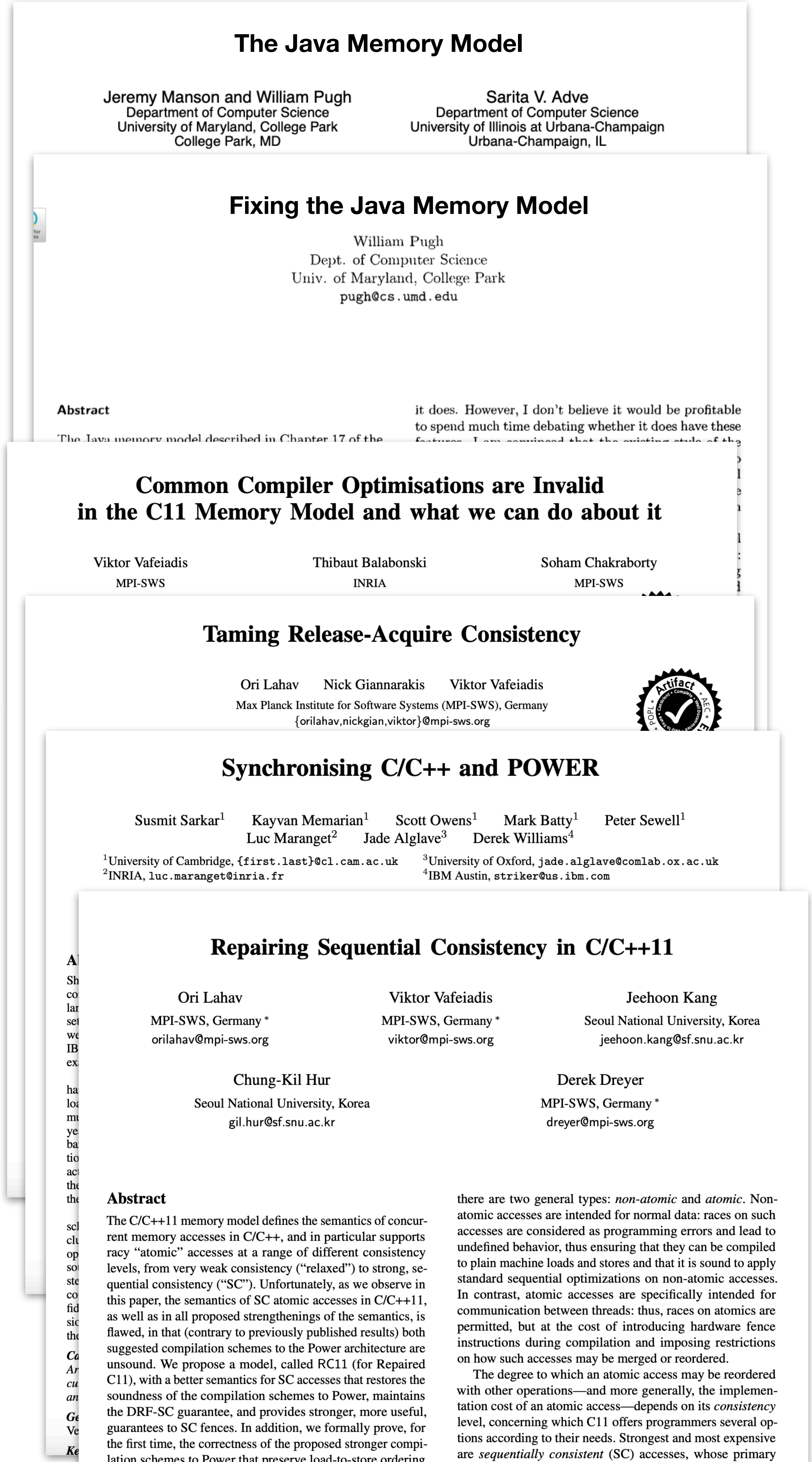

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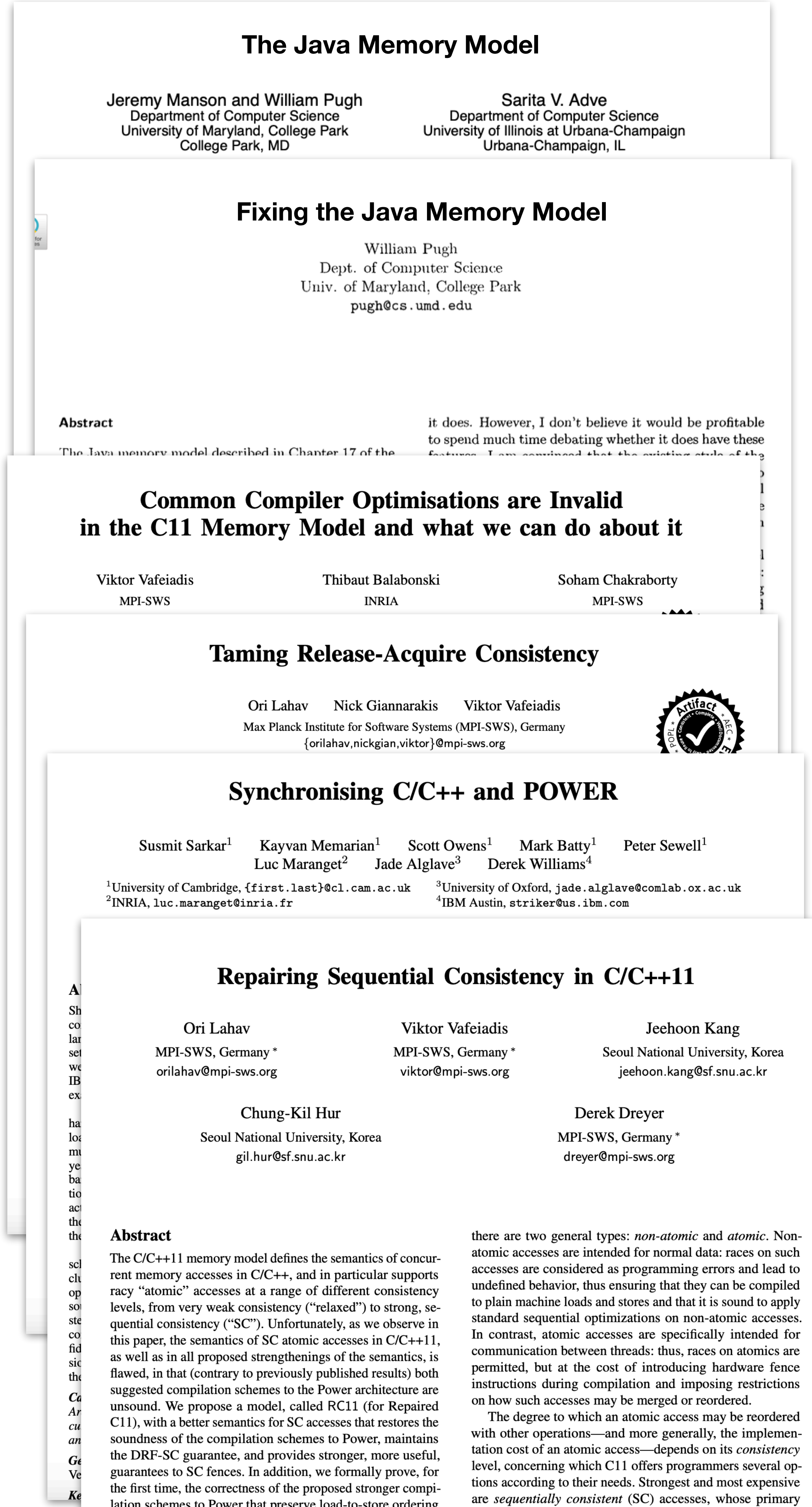
➡ **Need for automatic Memory Model verification!**



Dartagnan

Checking the Linux Kernel

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it was racy, failed to provide mutual exclusion, and could deadlock

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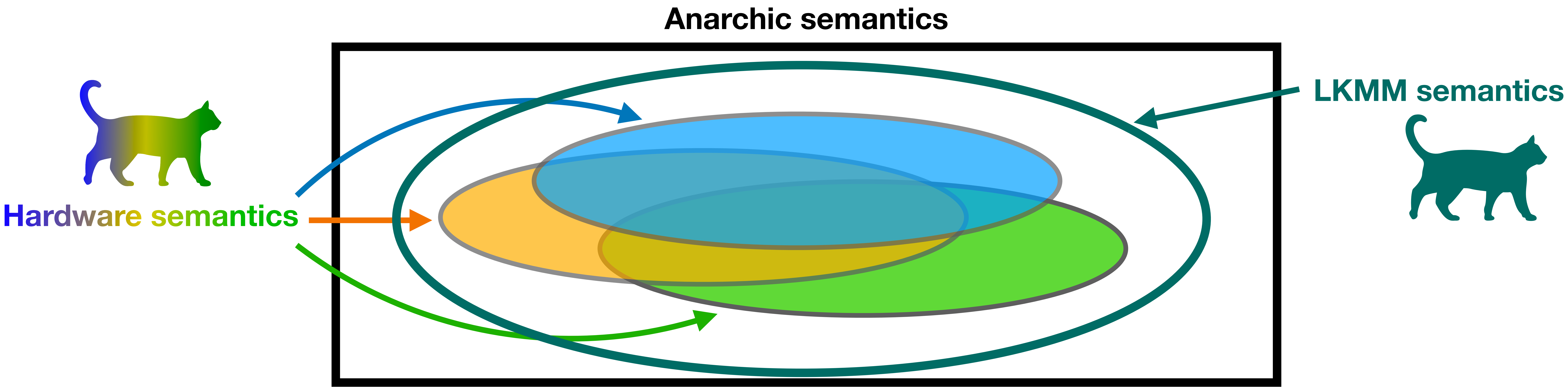

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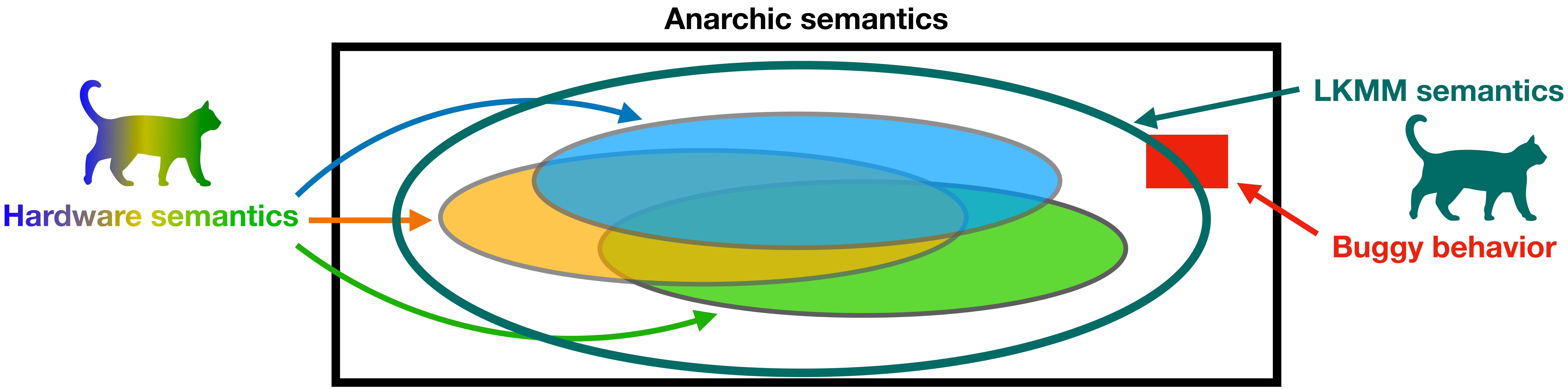

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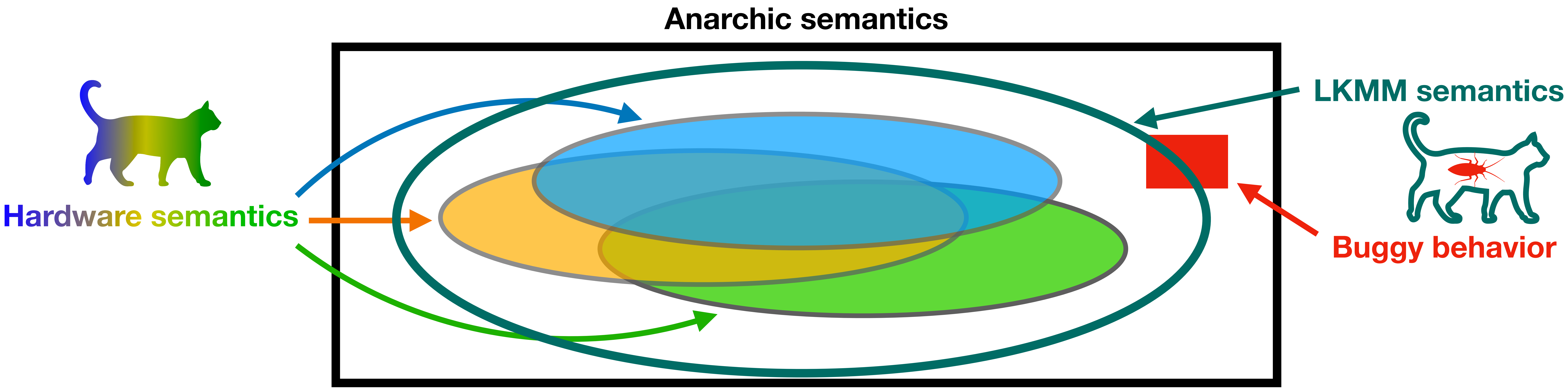

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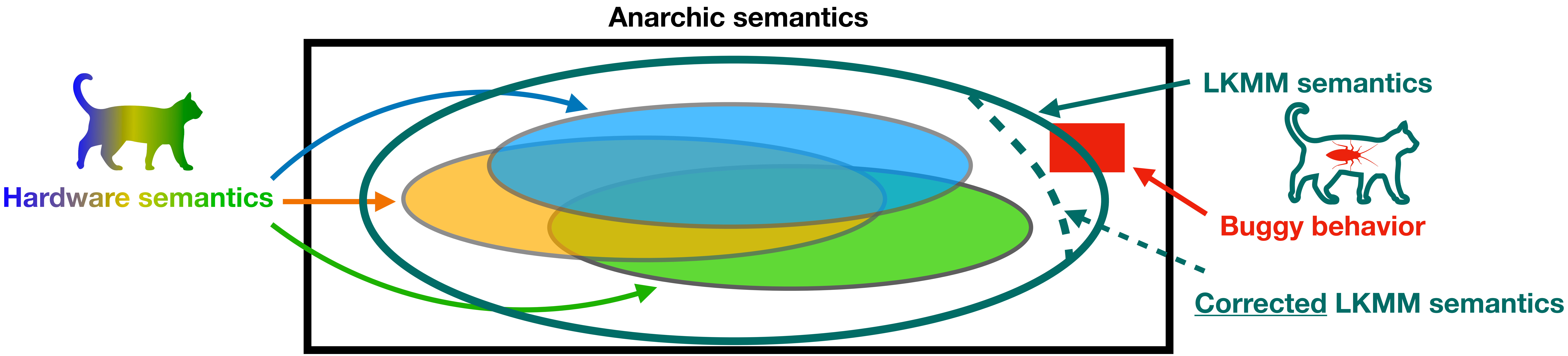

Model Checking Memory Models

Given: Memory Models M_1, M_2

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Relational $\top$,
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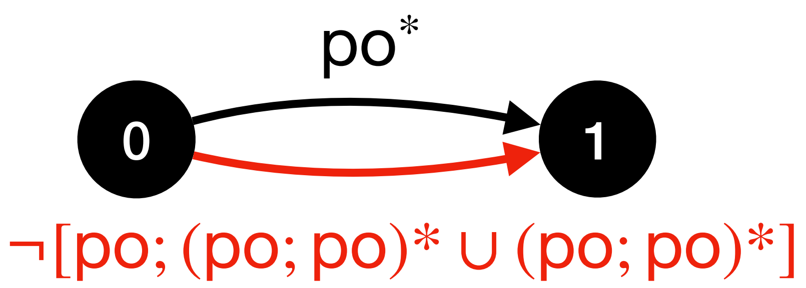

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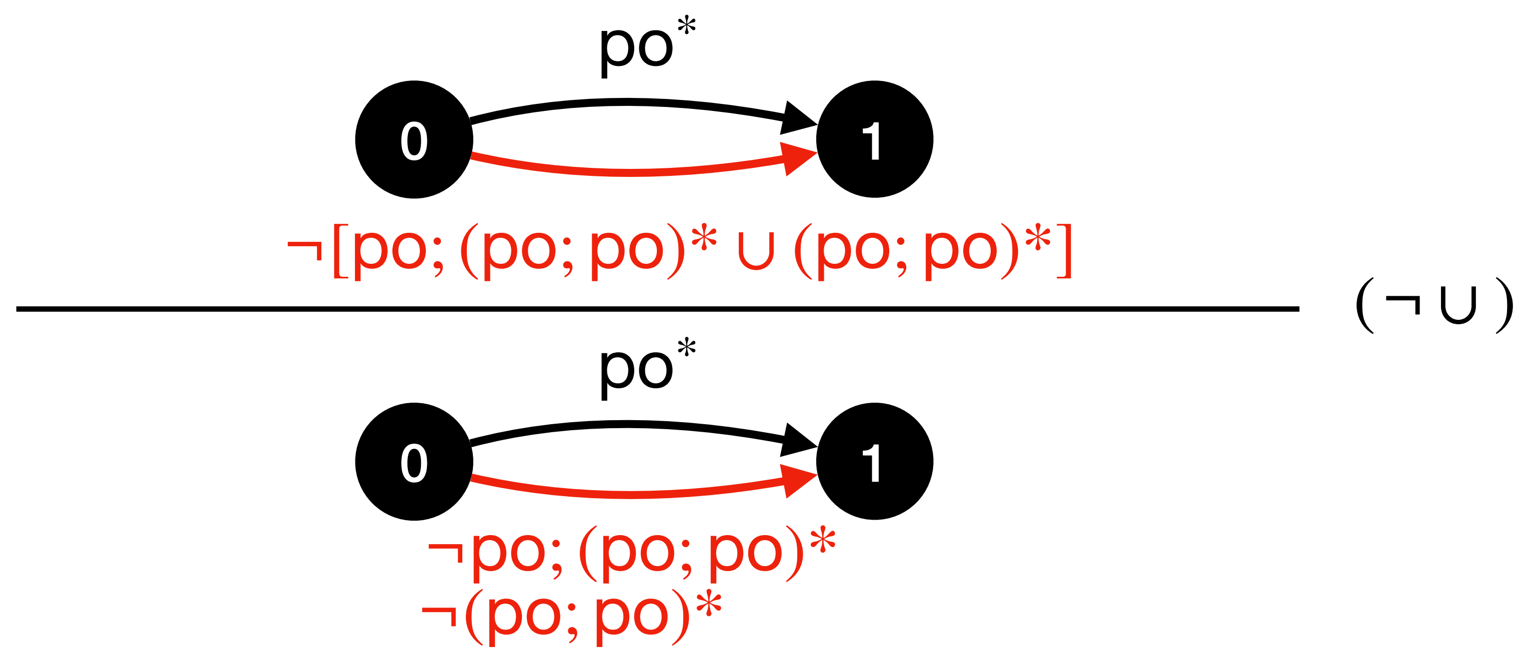

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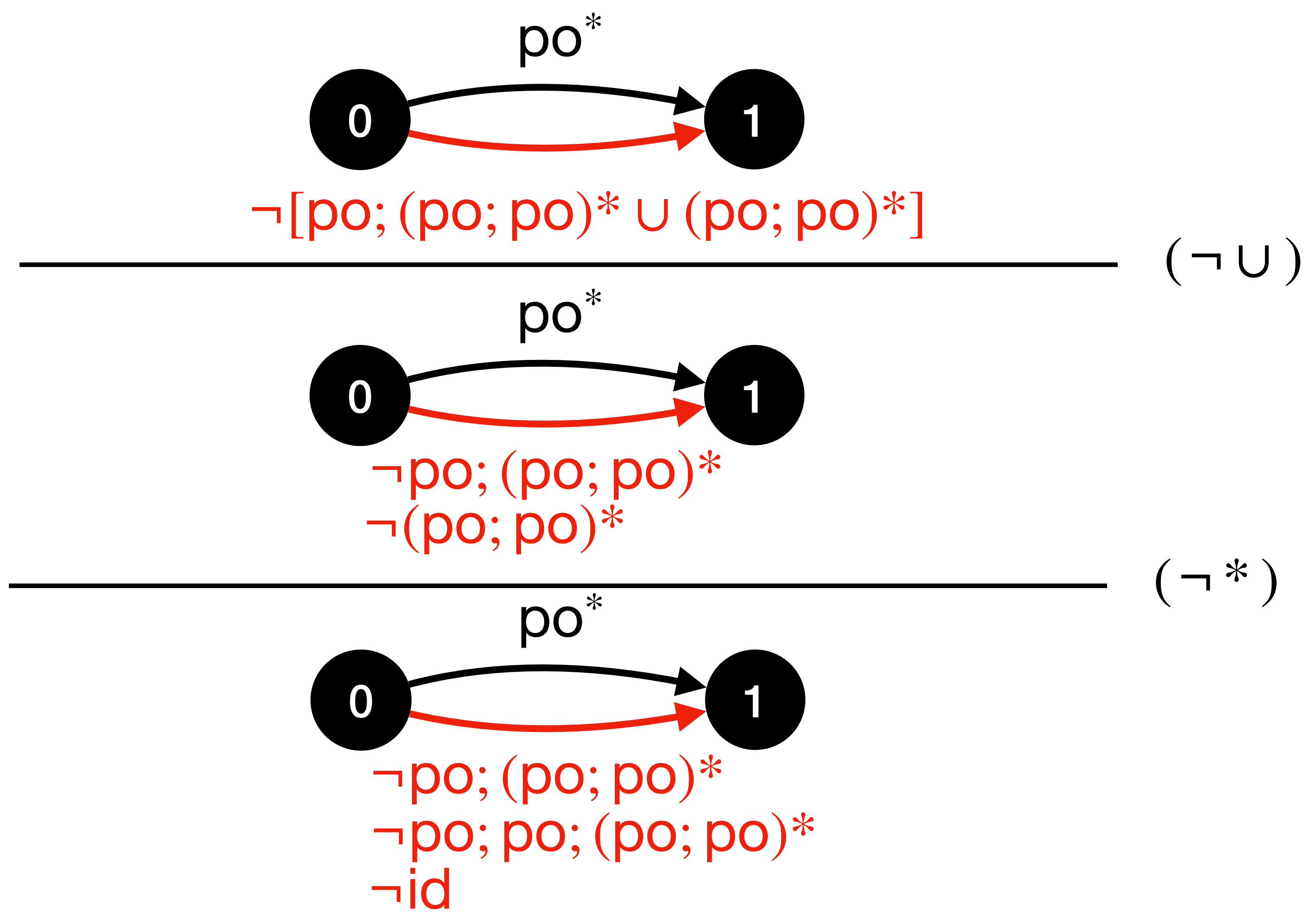

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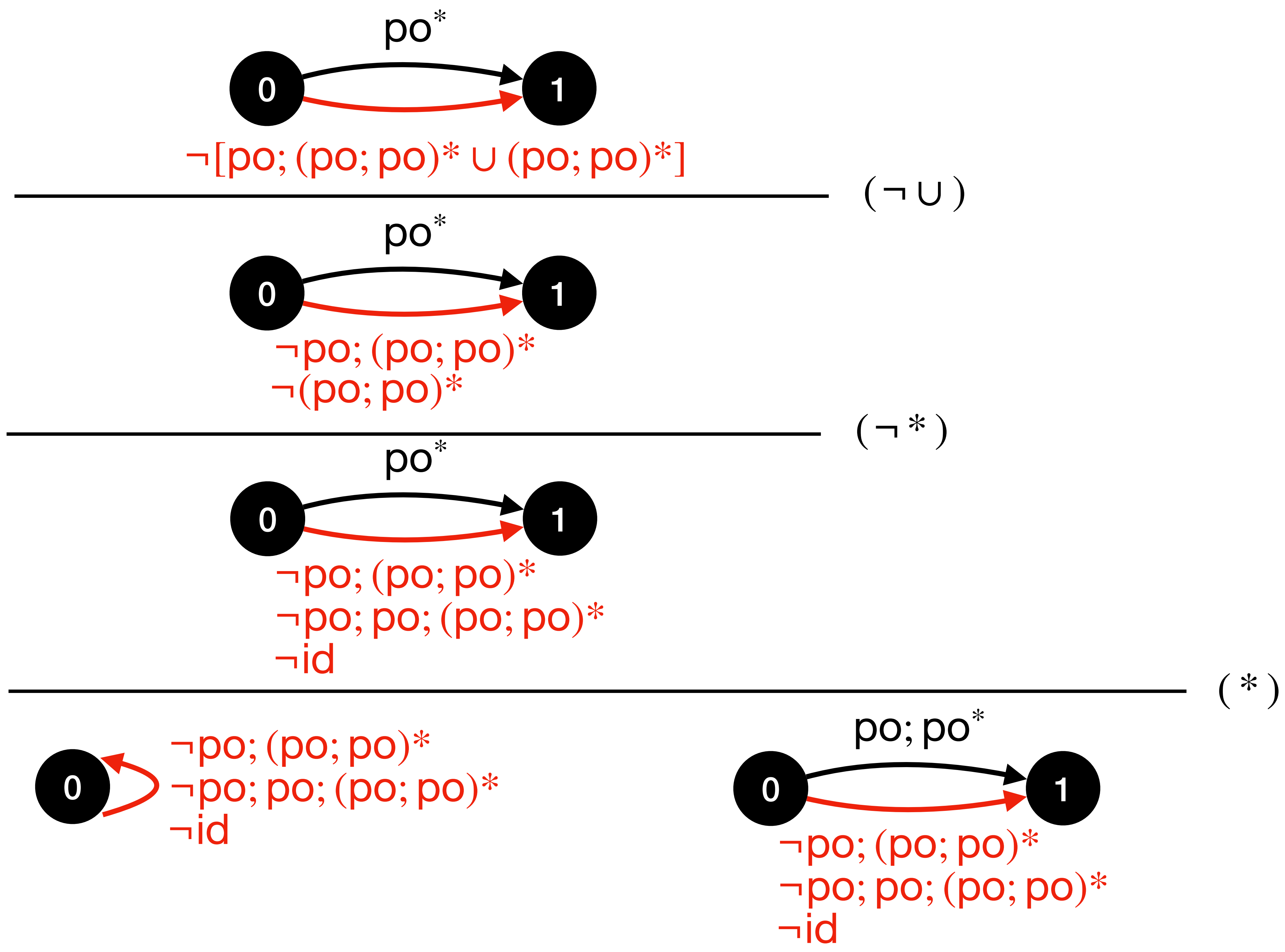

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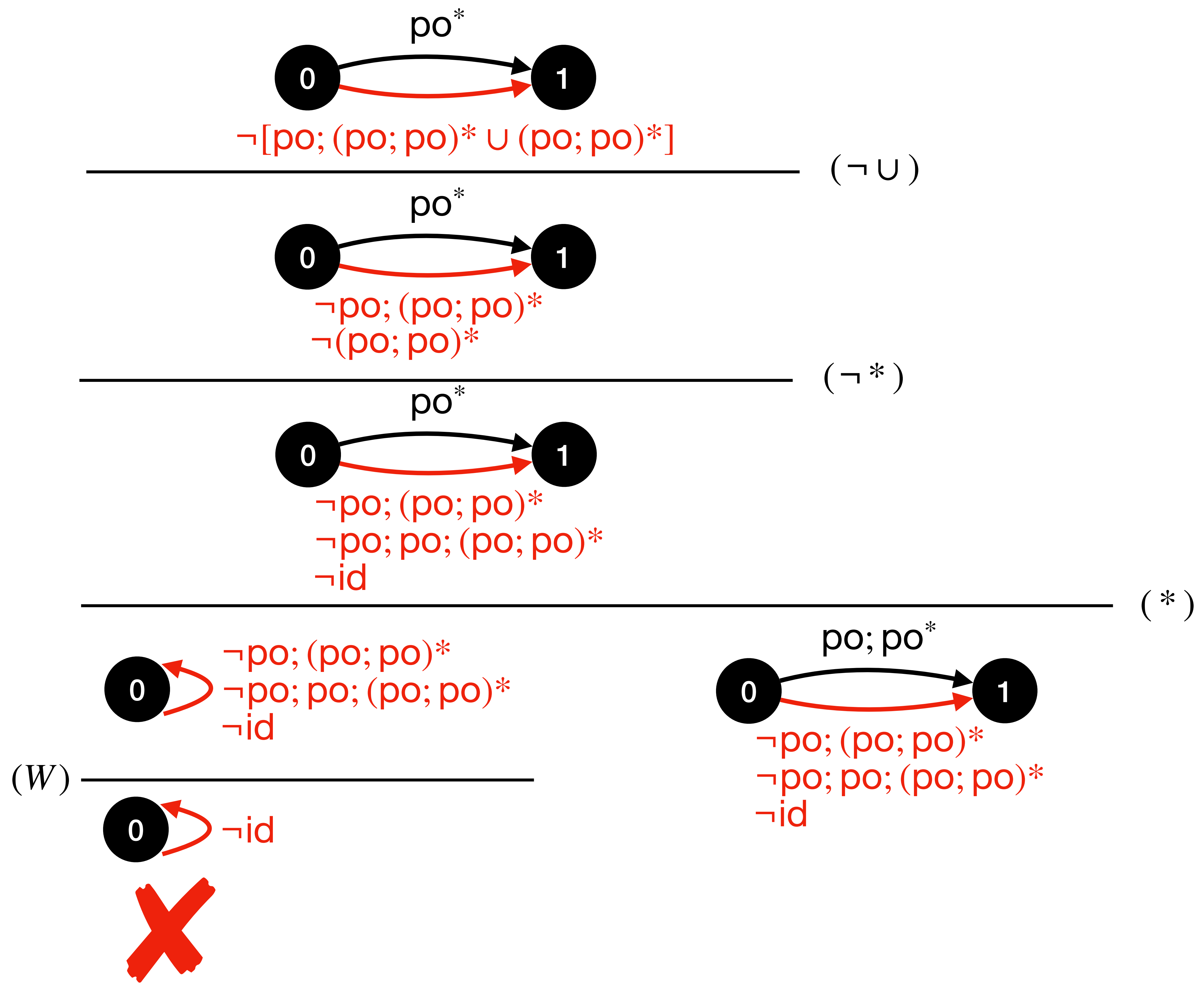

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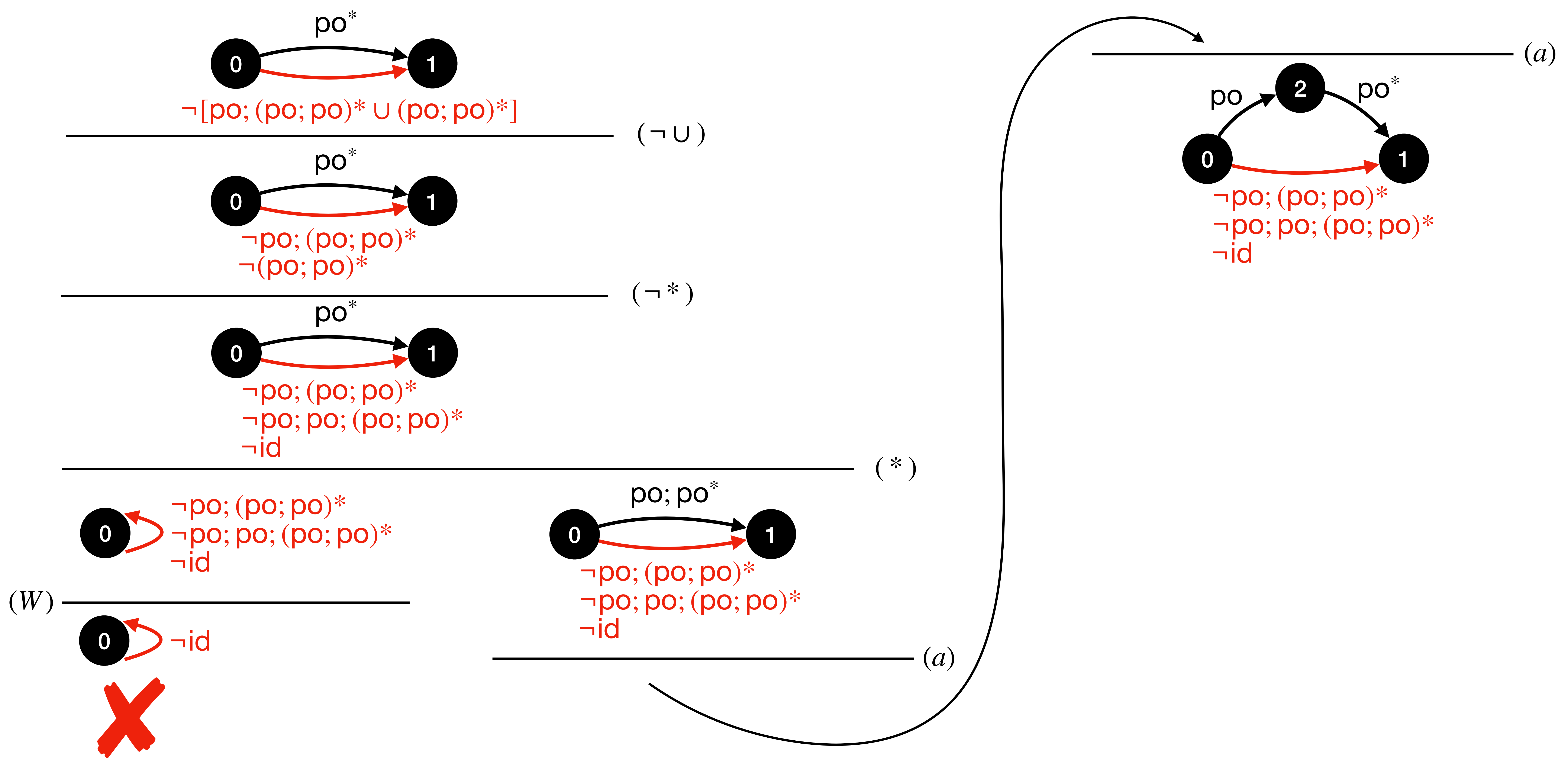

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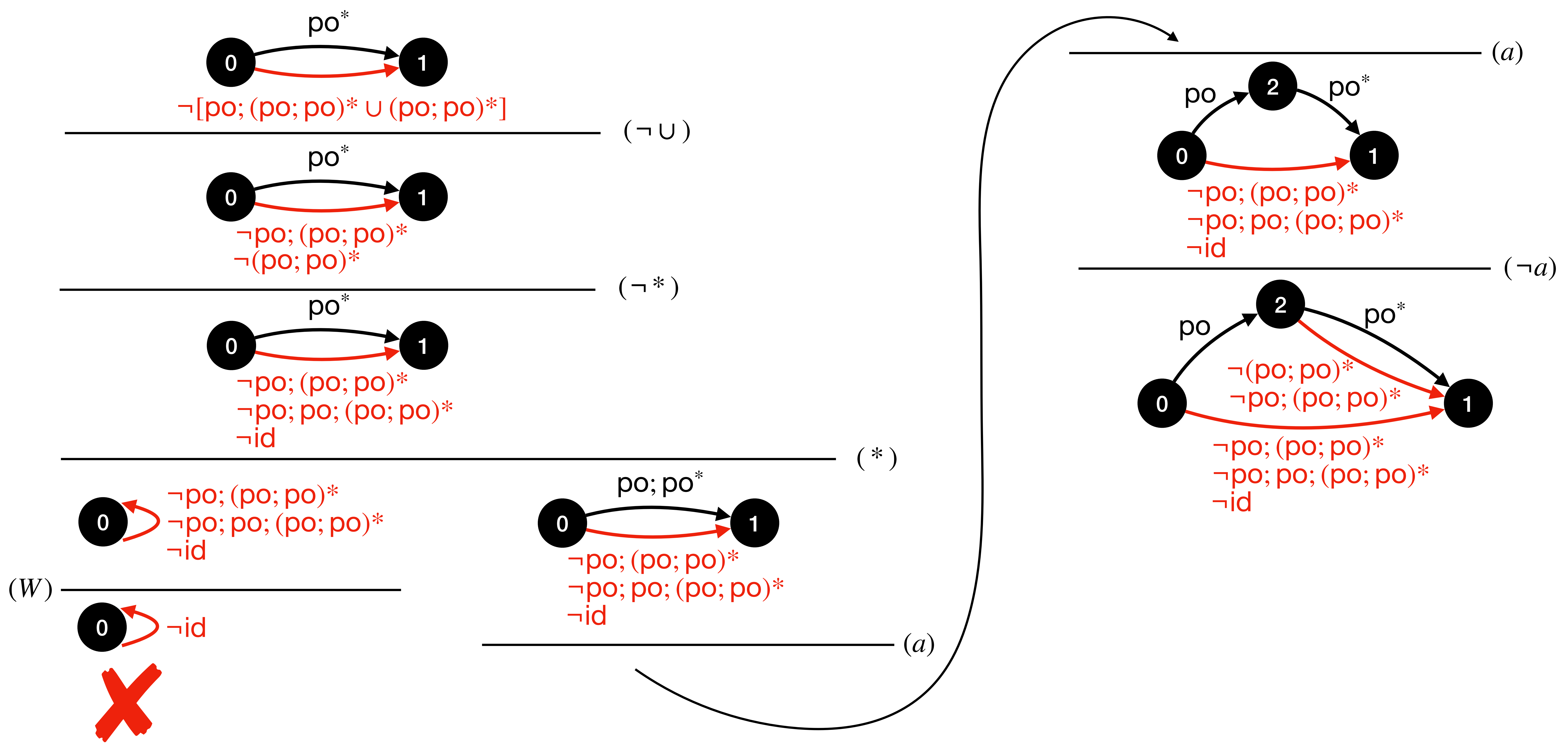

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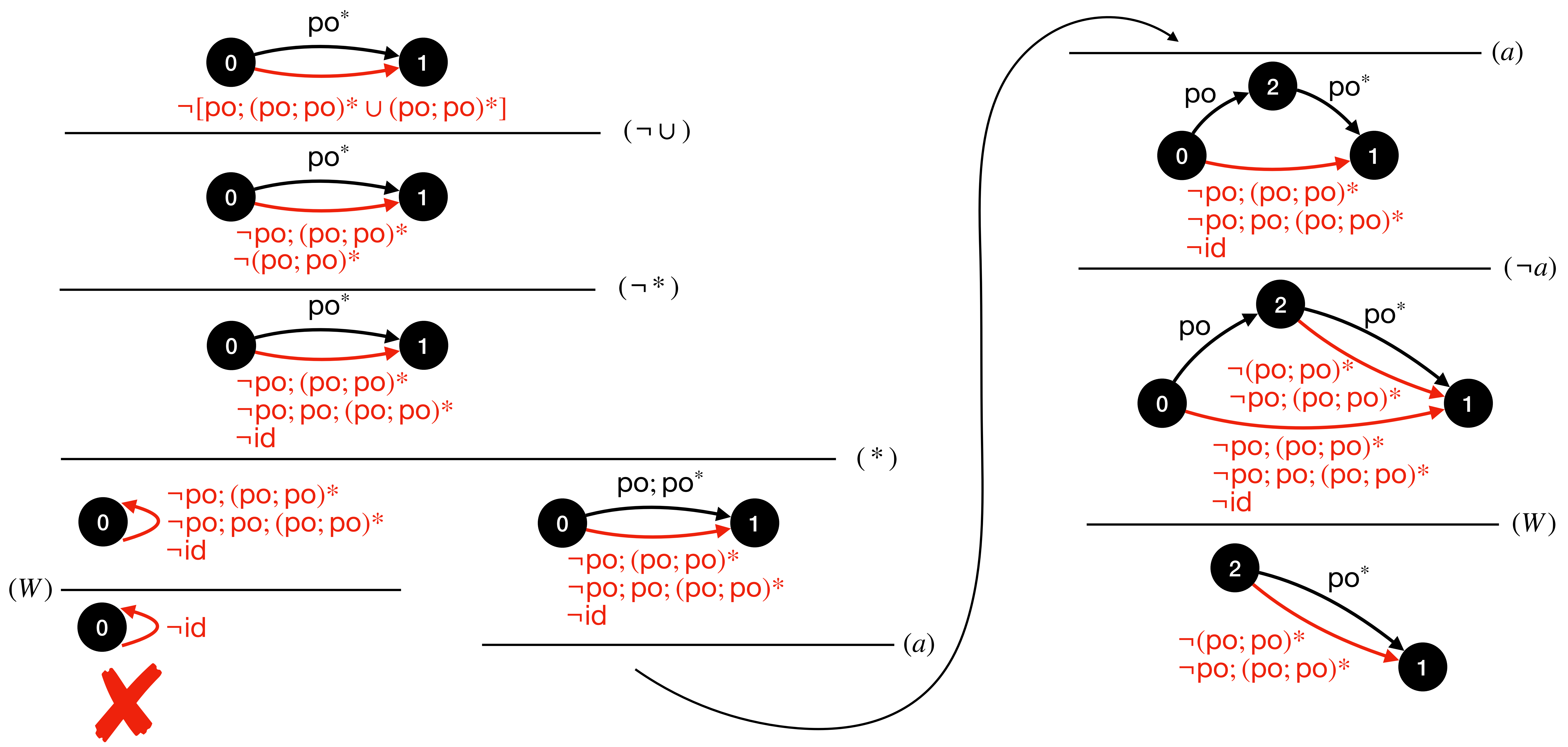

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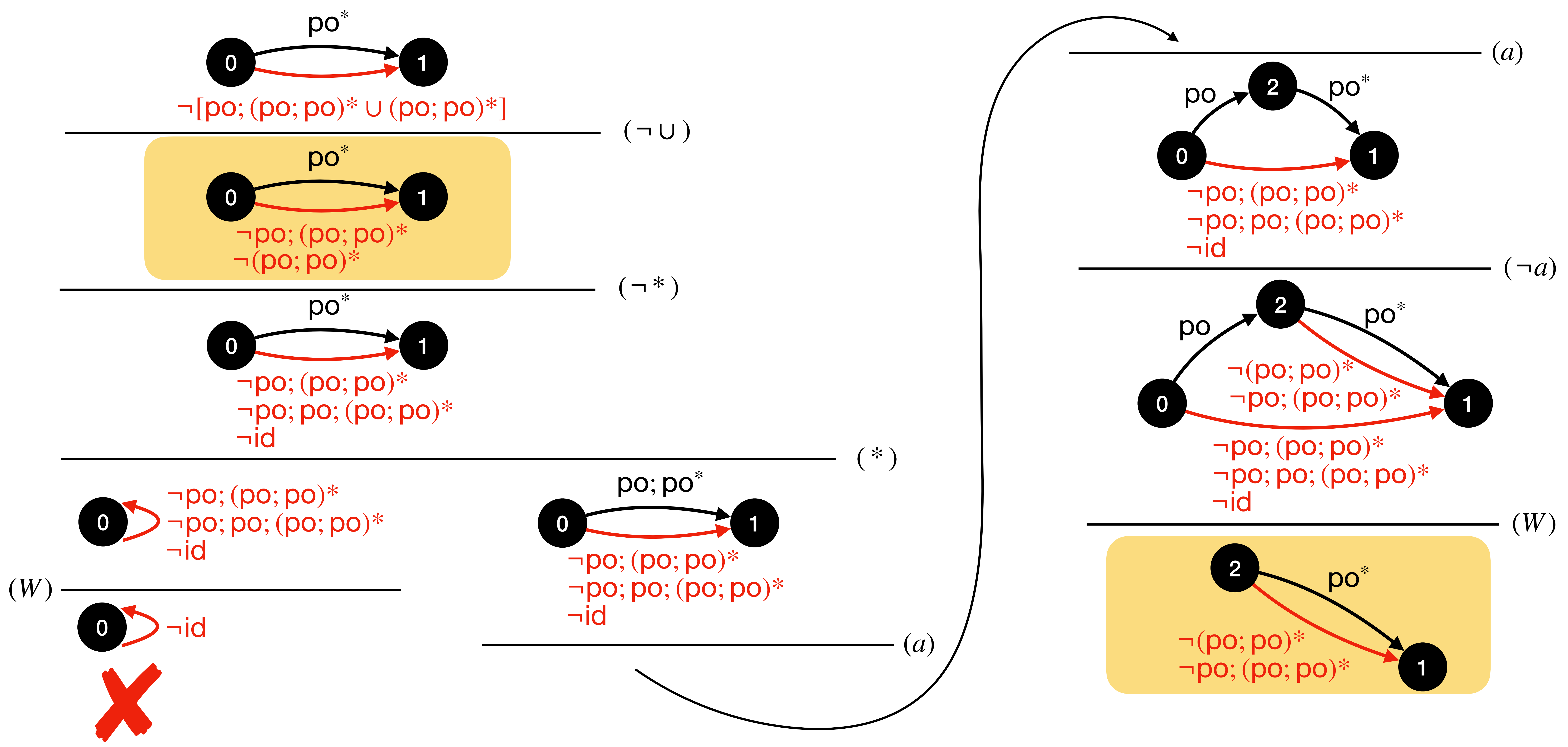

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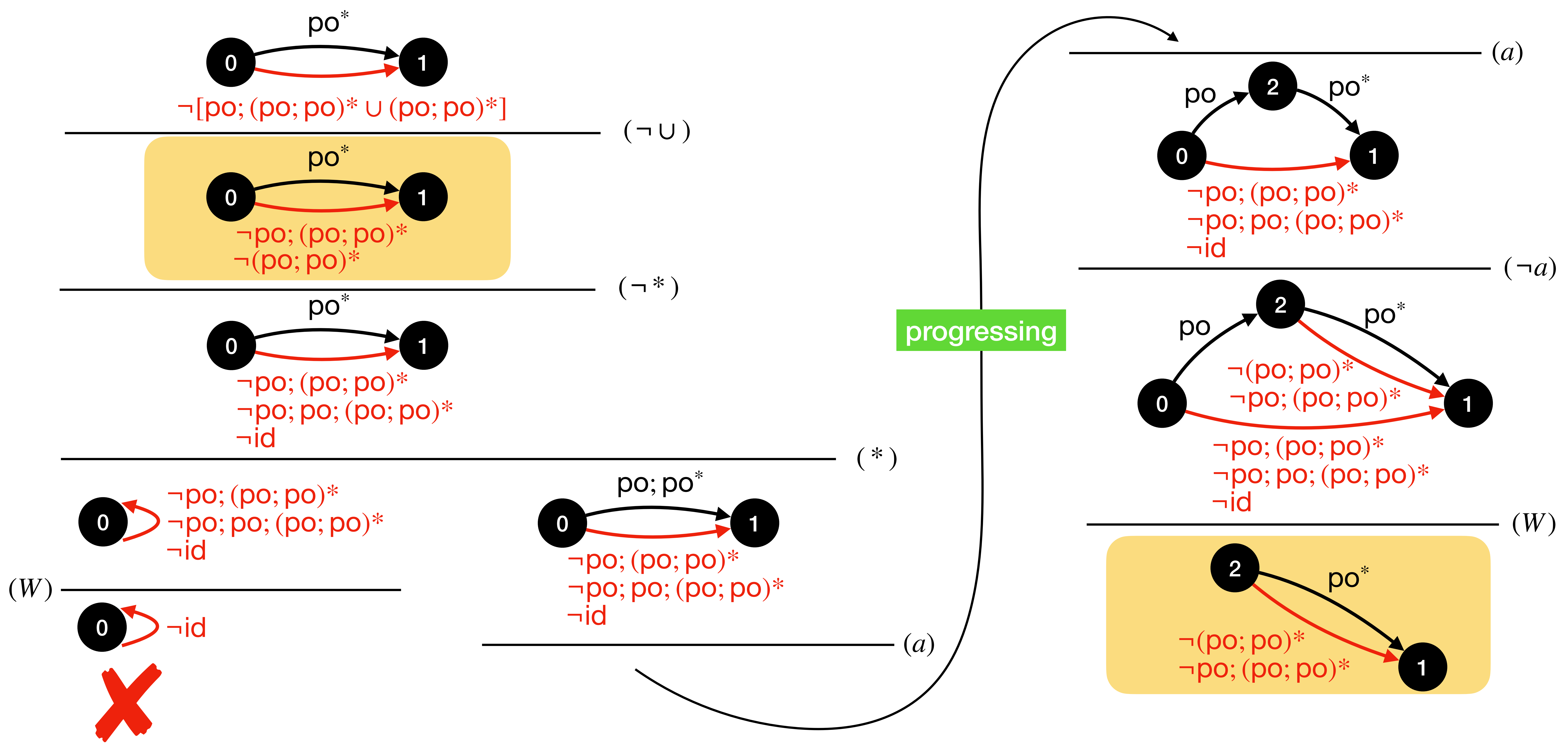
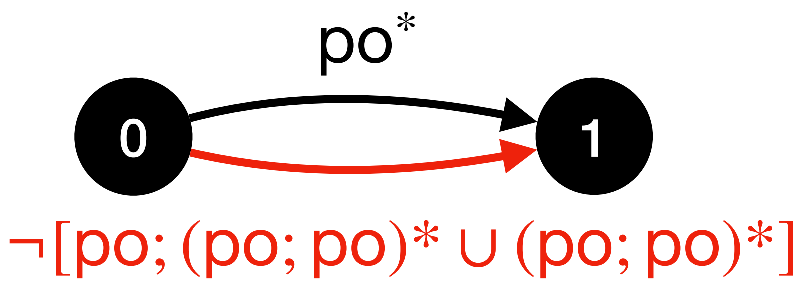


Cyclic Proof System

- Graphs = represented by relational algebra expressions + **event symbols**



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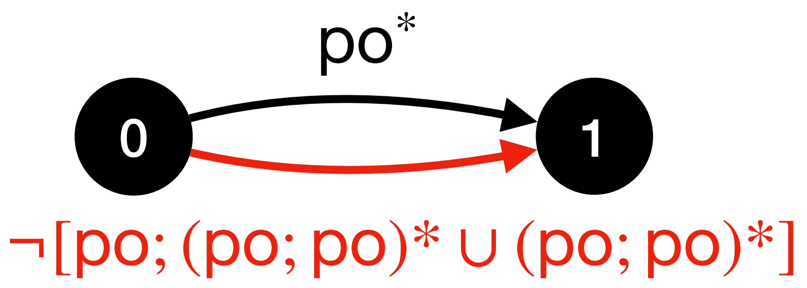

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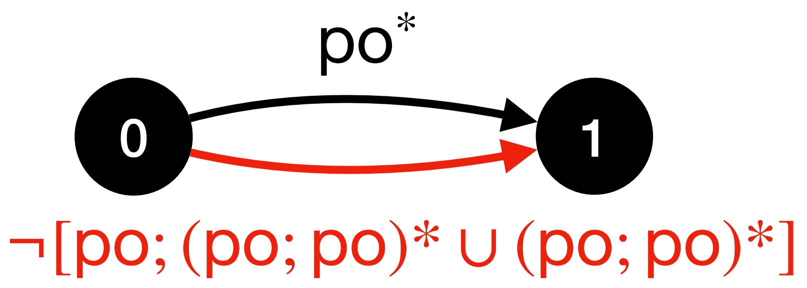

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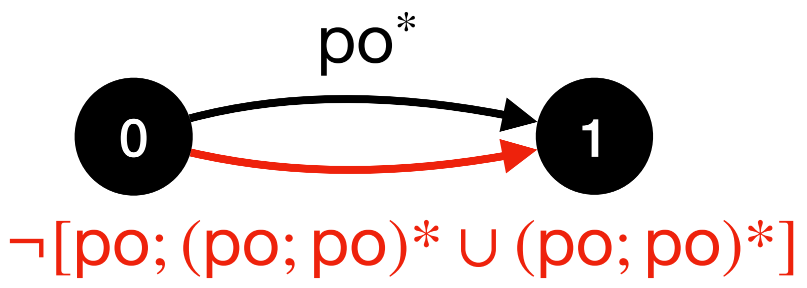

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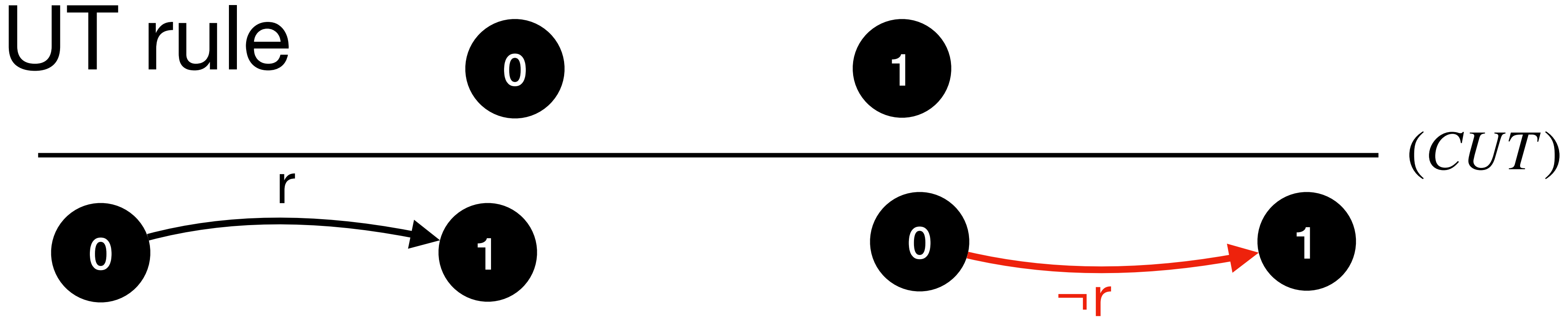

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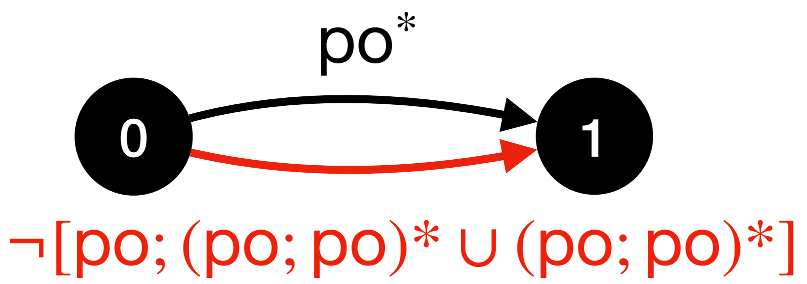

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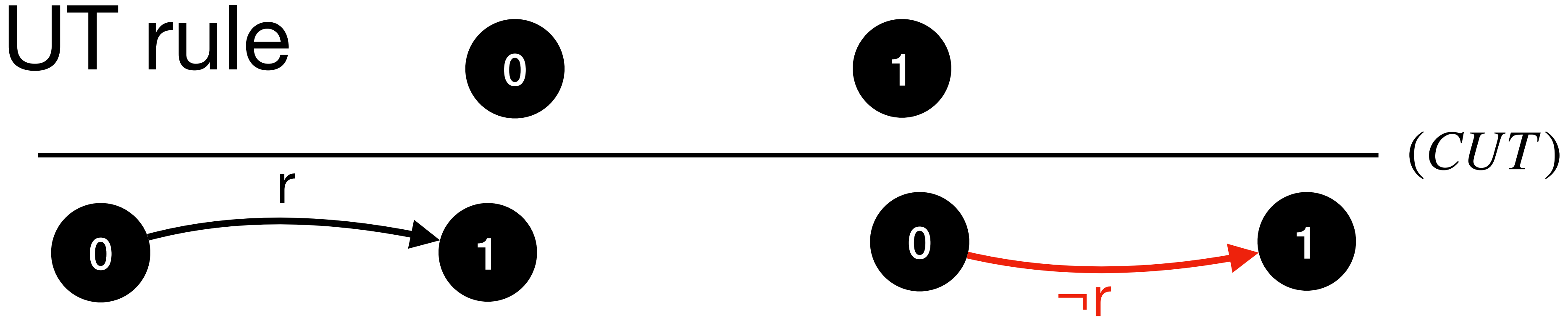


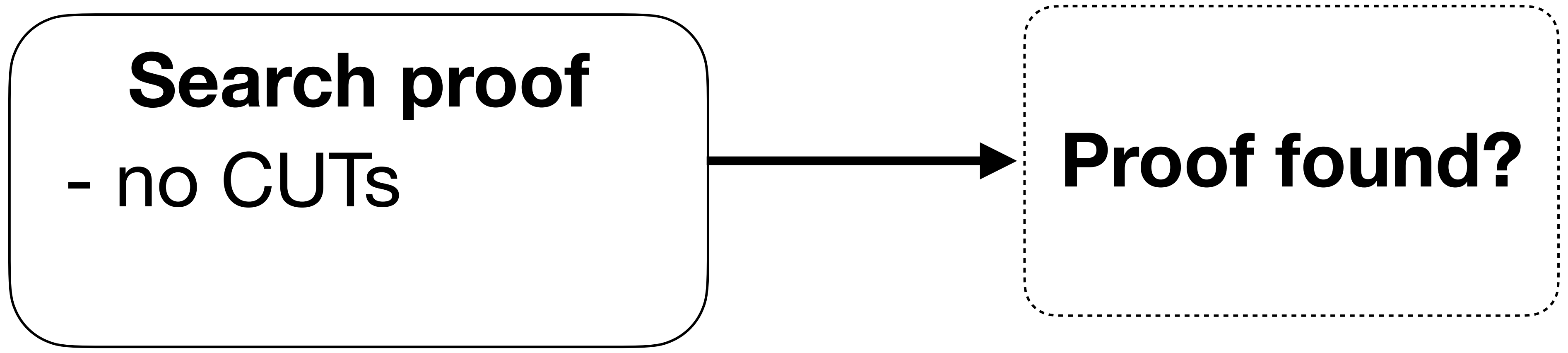
➡ **Naive proof search is inefficient (EXPSPACE-complete)**

CEGAR Proof Search

Search proof

- no CUTs

CEGAR Proof Search



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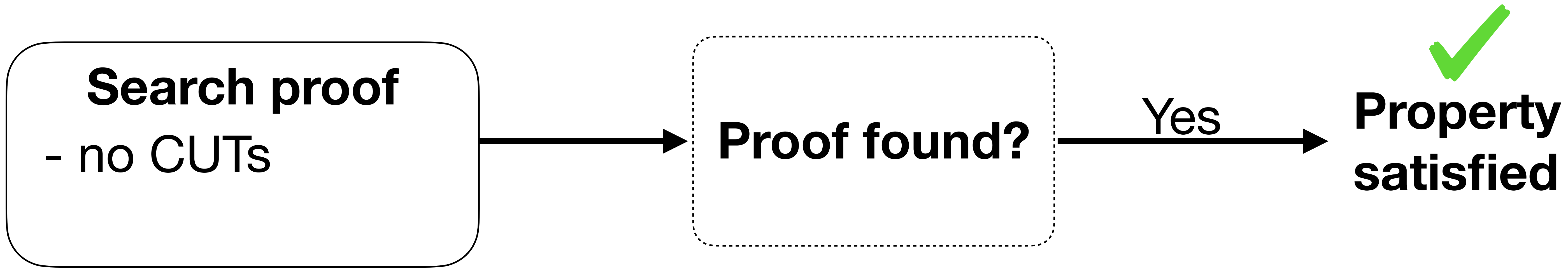

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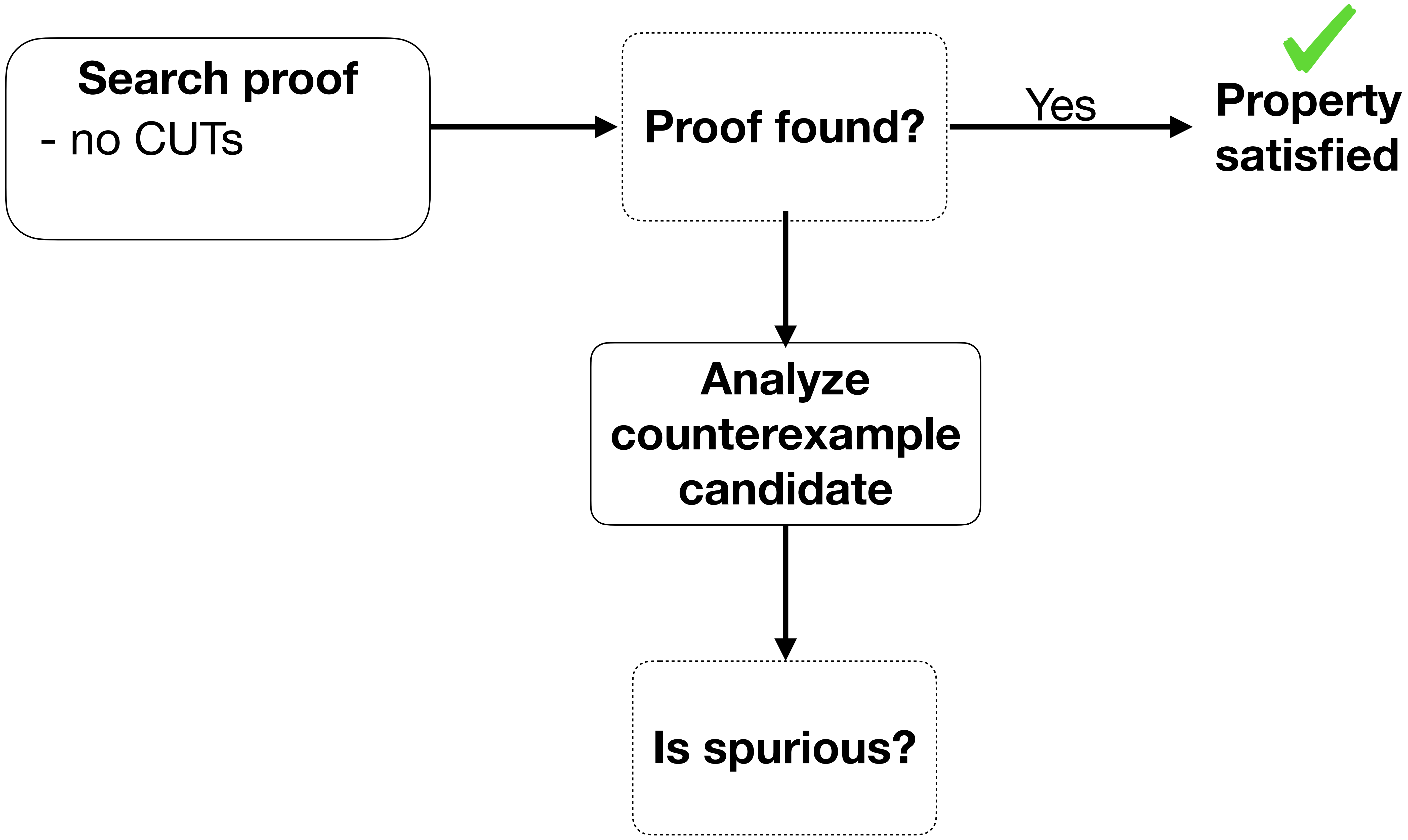

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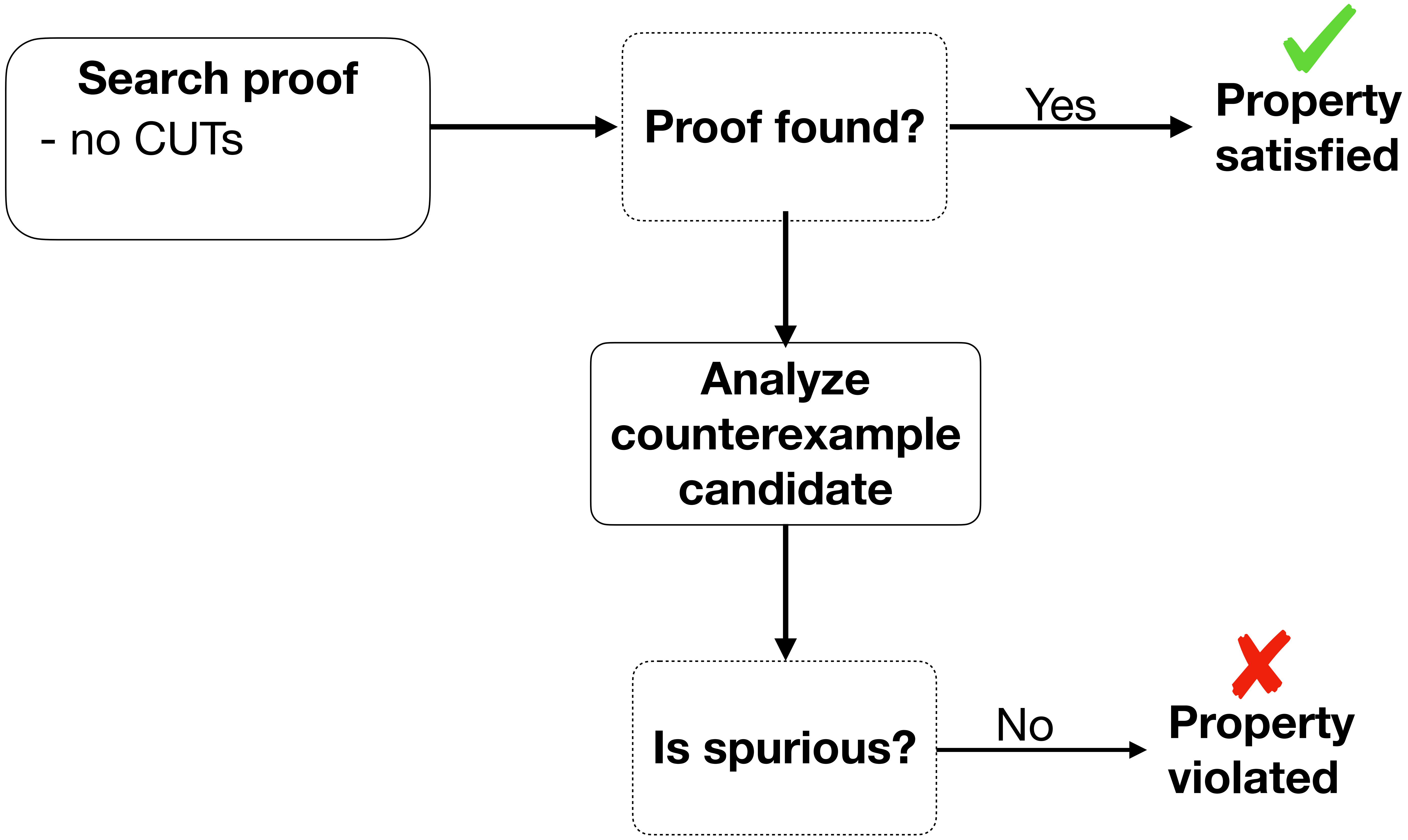

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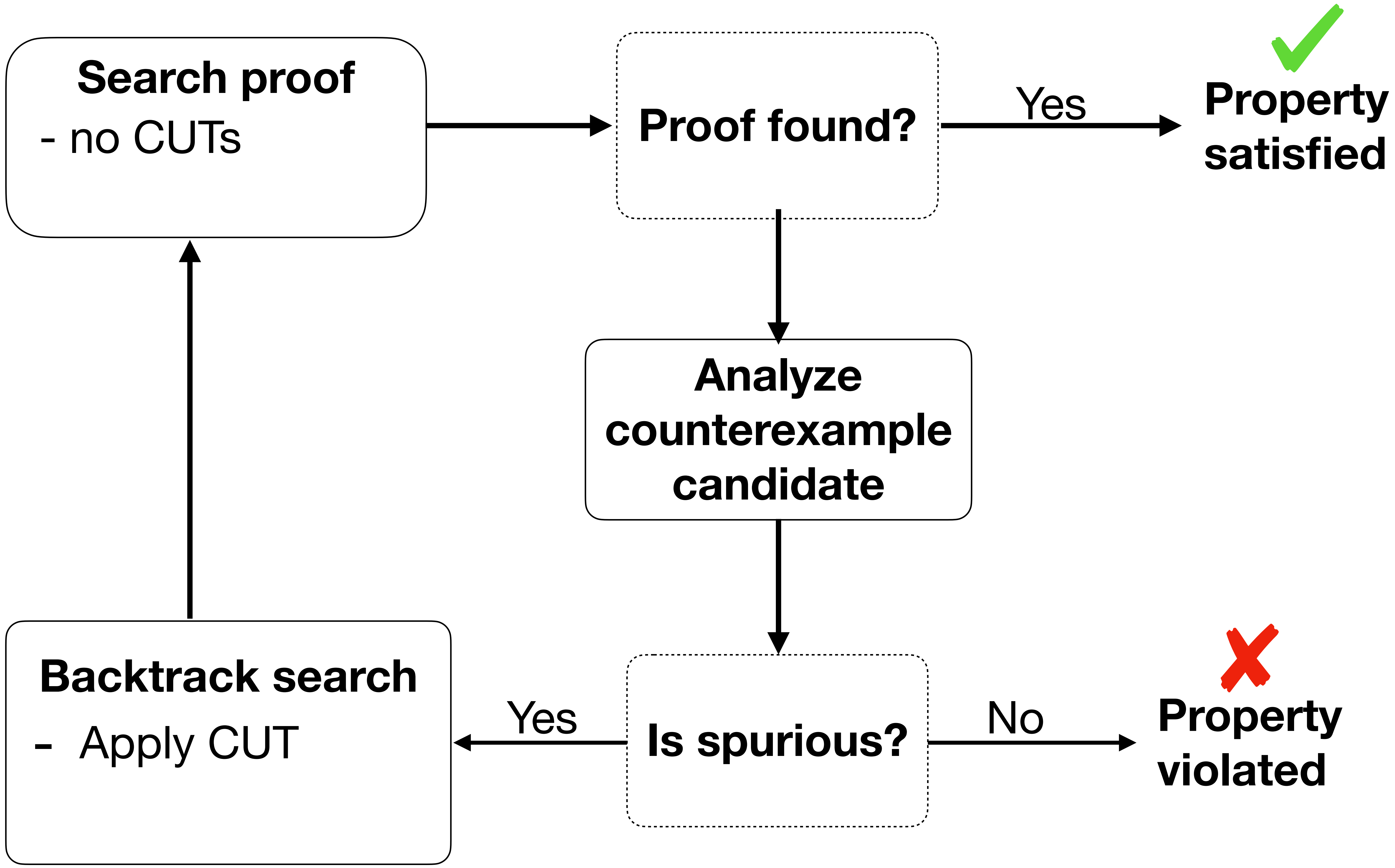

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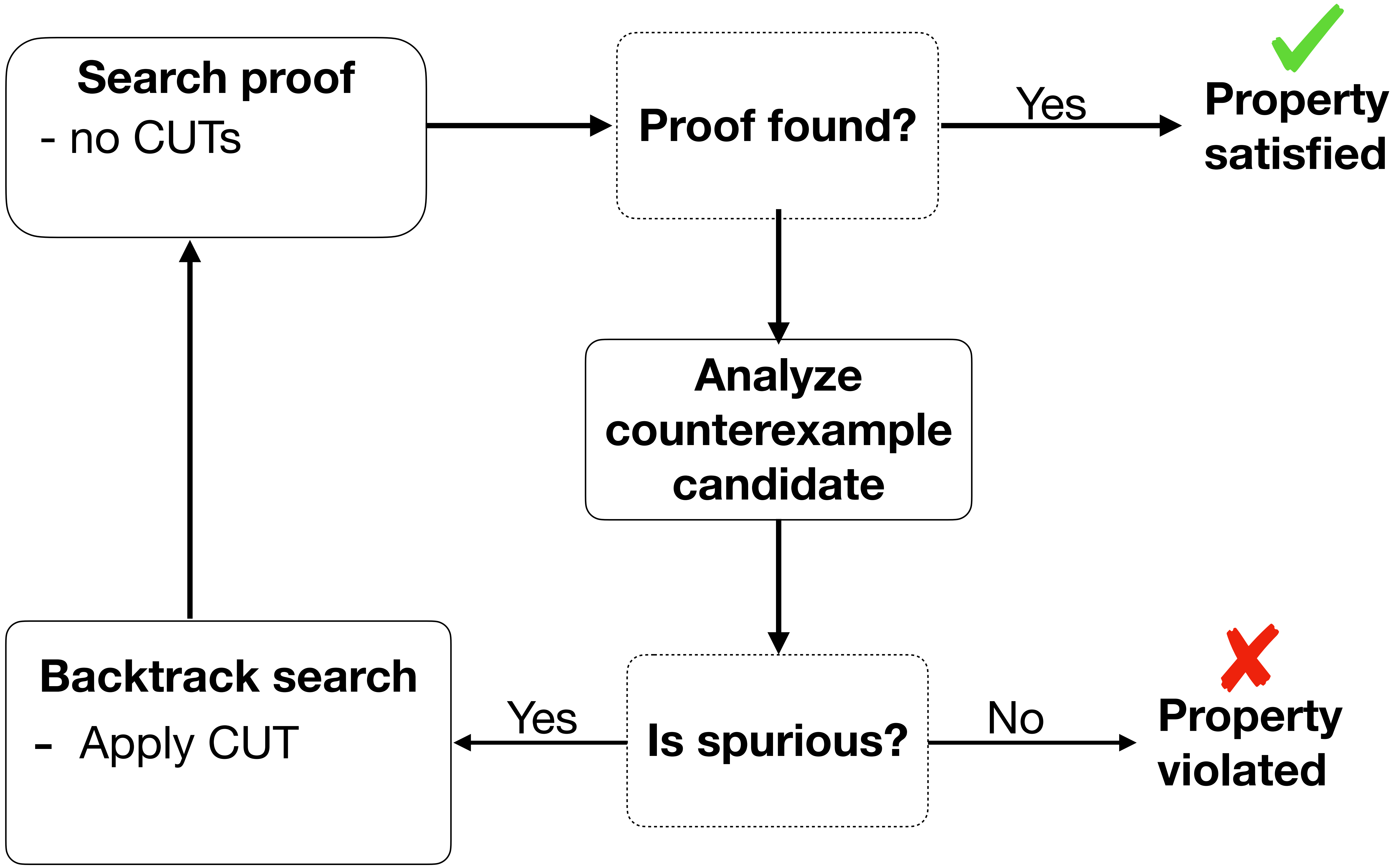

Problem: checking inclusions in relational algebra is not sufficient

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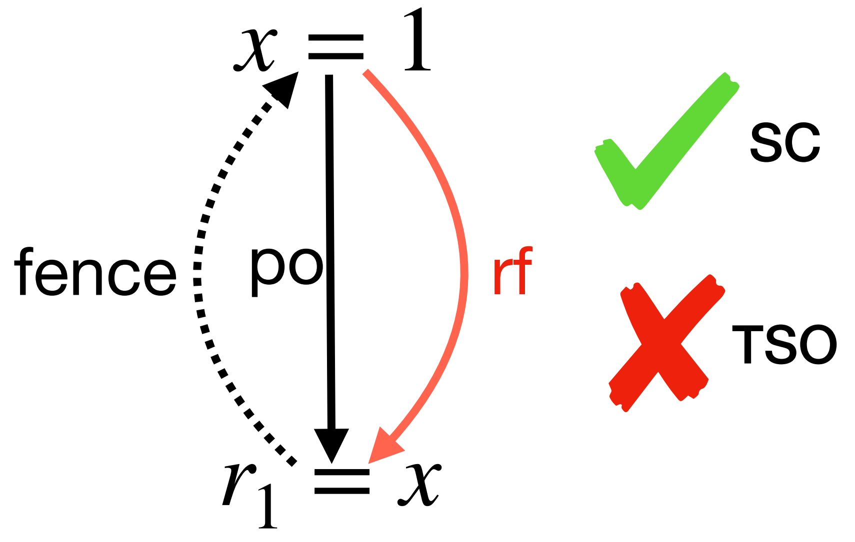

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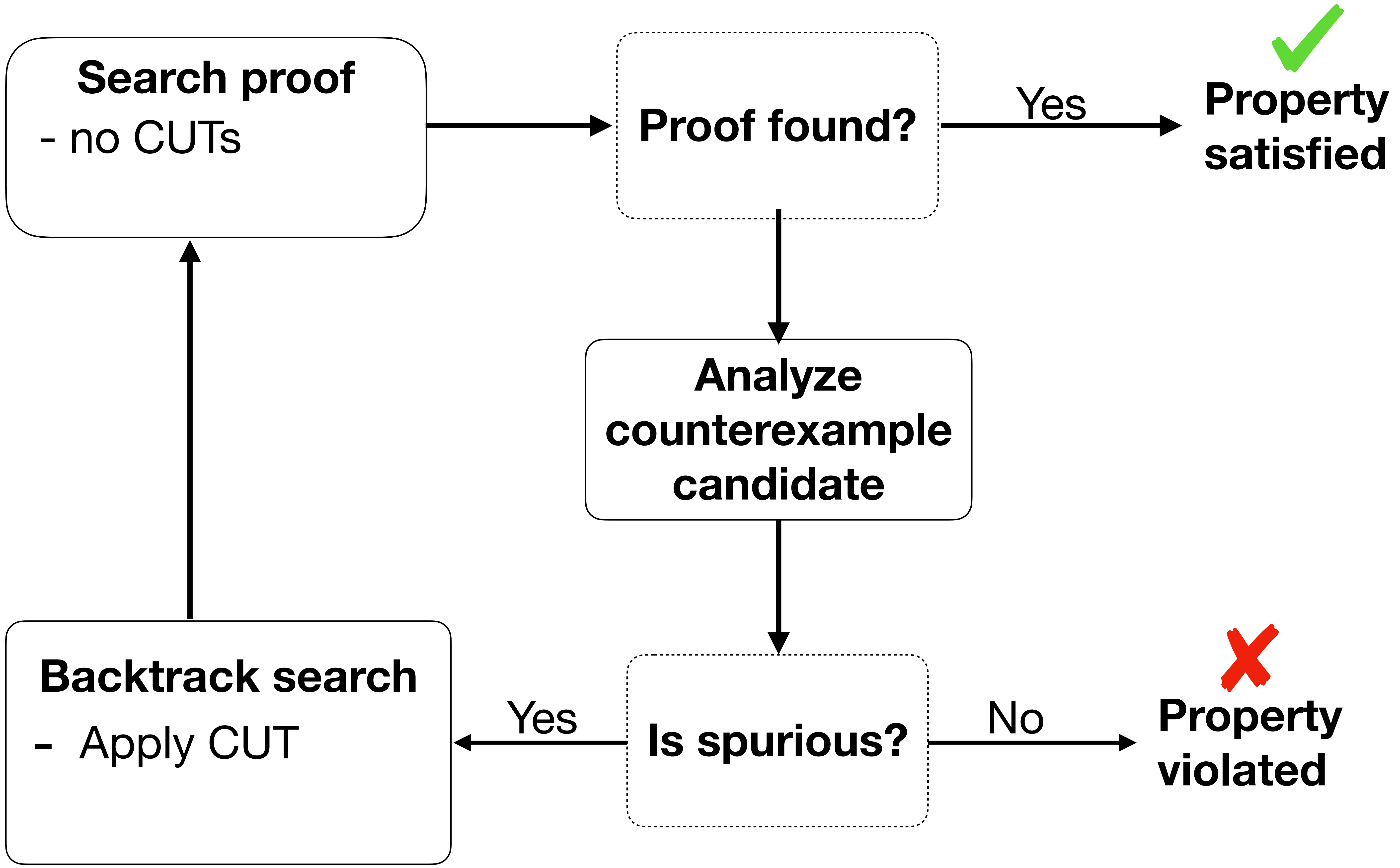

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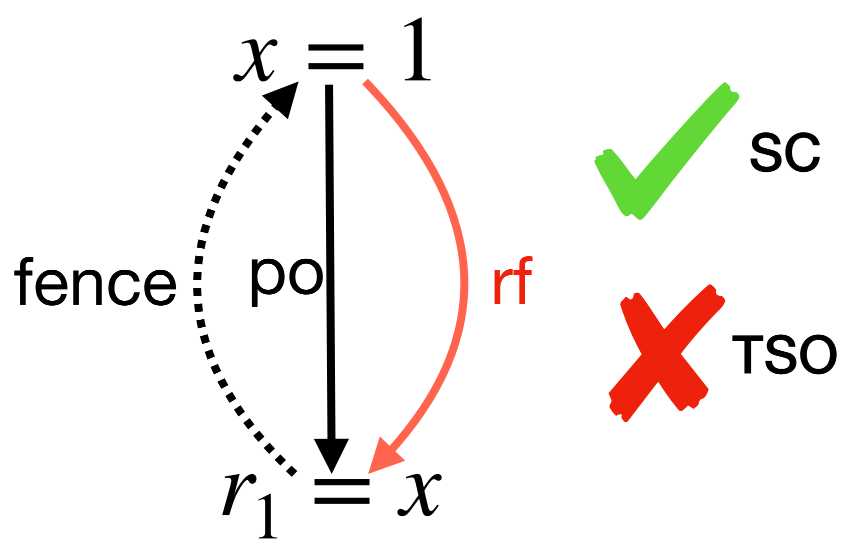

Solution: use assumptions to restrict graphs to executions

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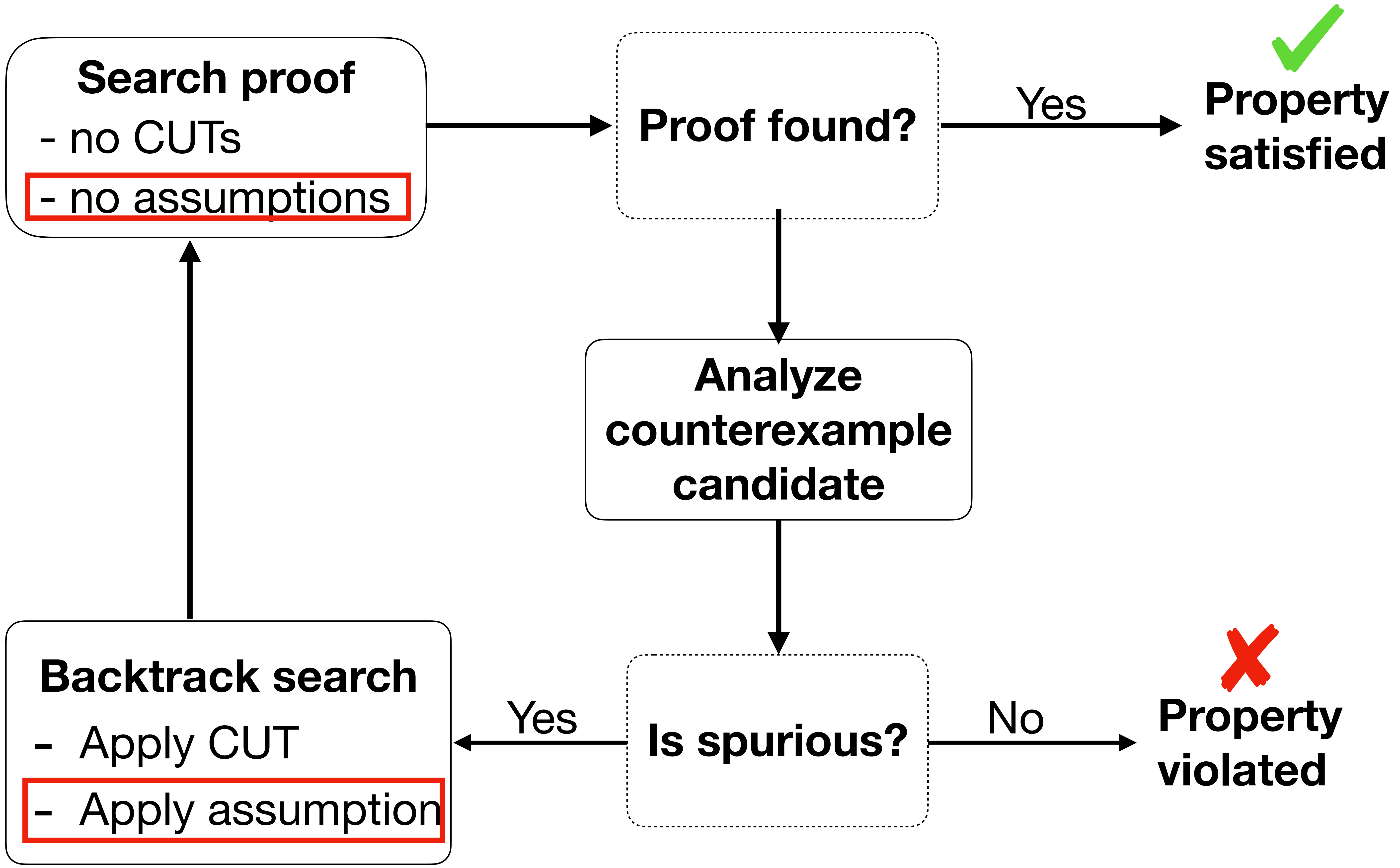
CEGAR Proof Search



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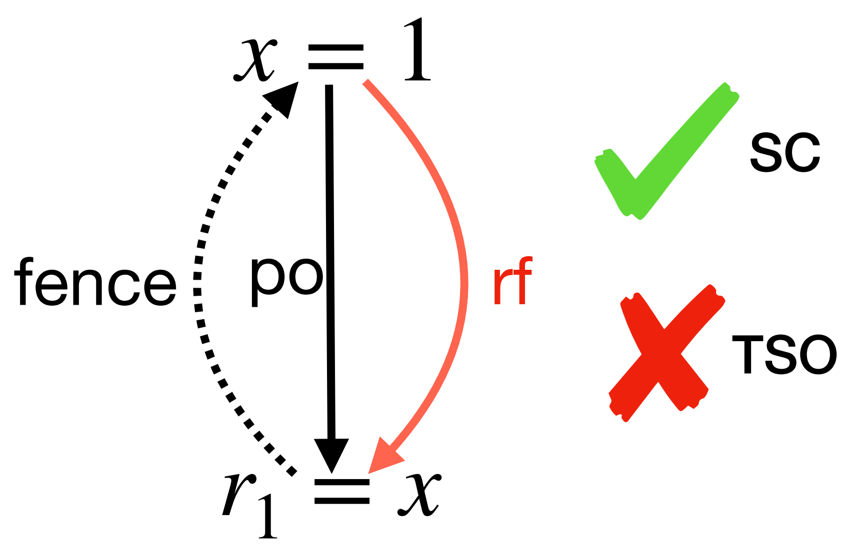

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Hard Problems in Verification

Complexity of VASS Reachability

Decidability of Regular Separability for VASS Reachability Languages

Decidability of PVASS Reachability

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Complexity of Parity Games

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Regular Separability of VASS Reachability Languages

Eren Keskin, Roland Meyer: On the separability problem of VASS reachability languages @ LICS24

Regular Separability

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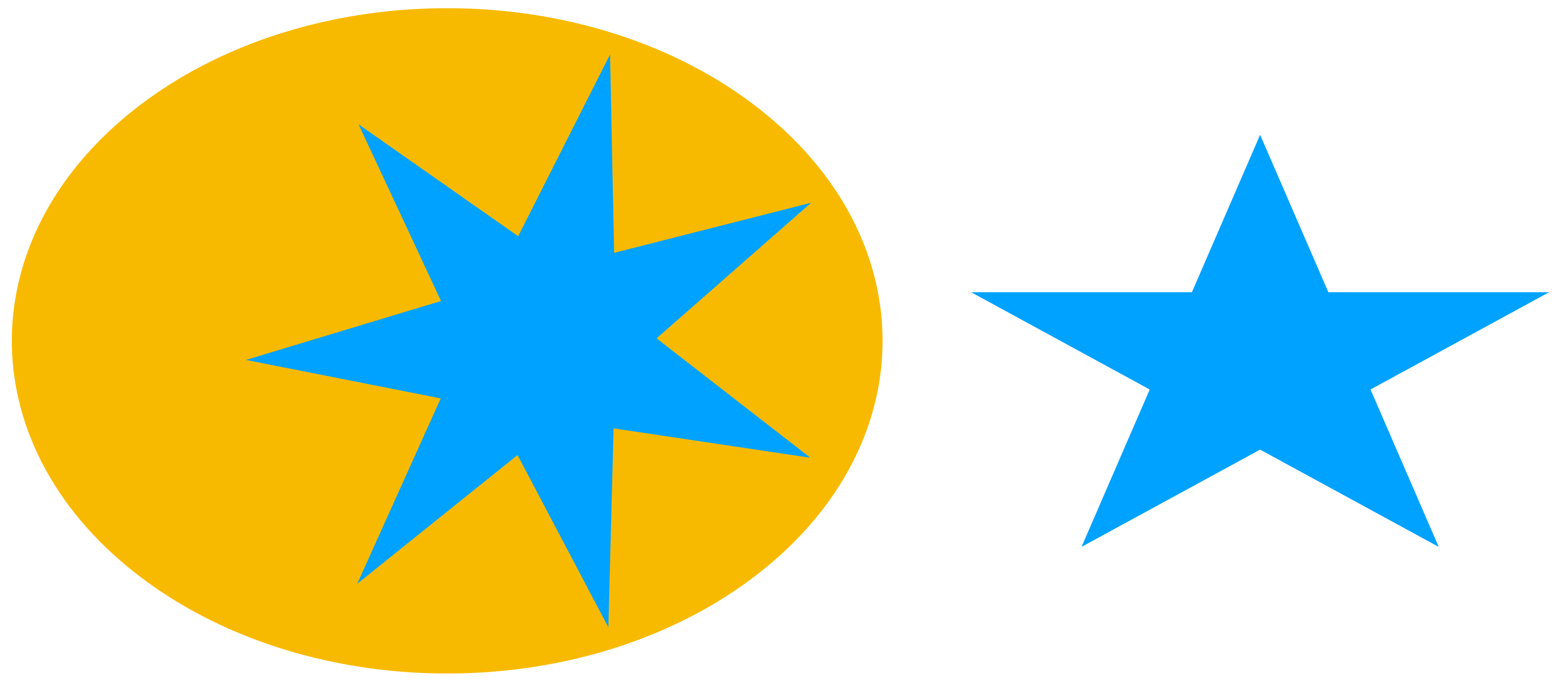

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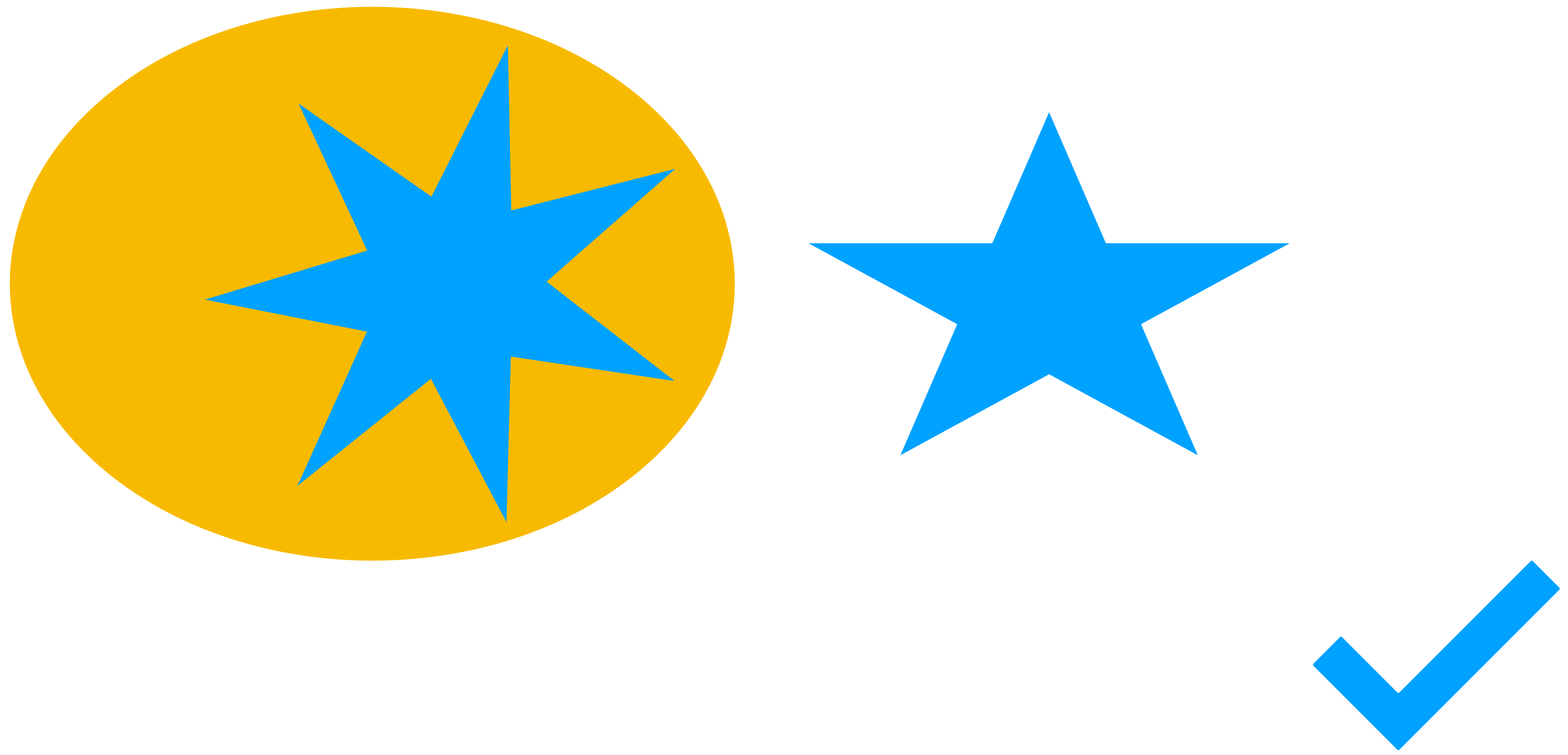

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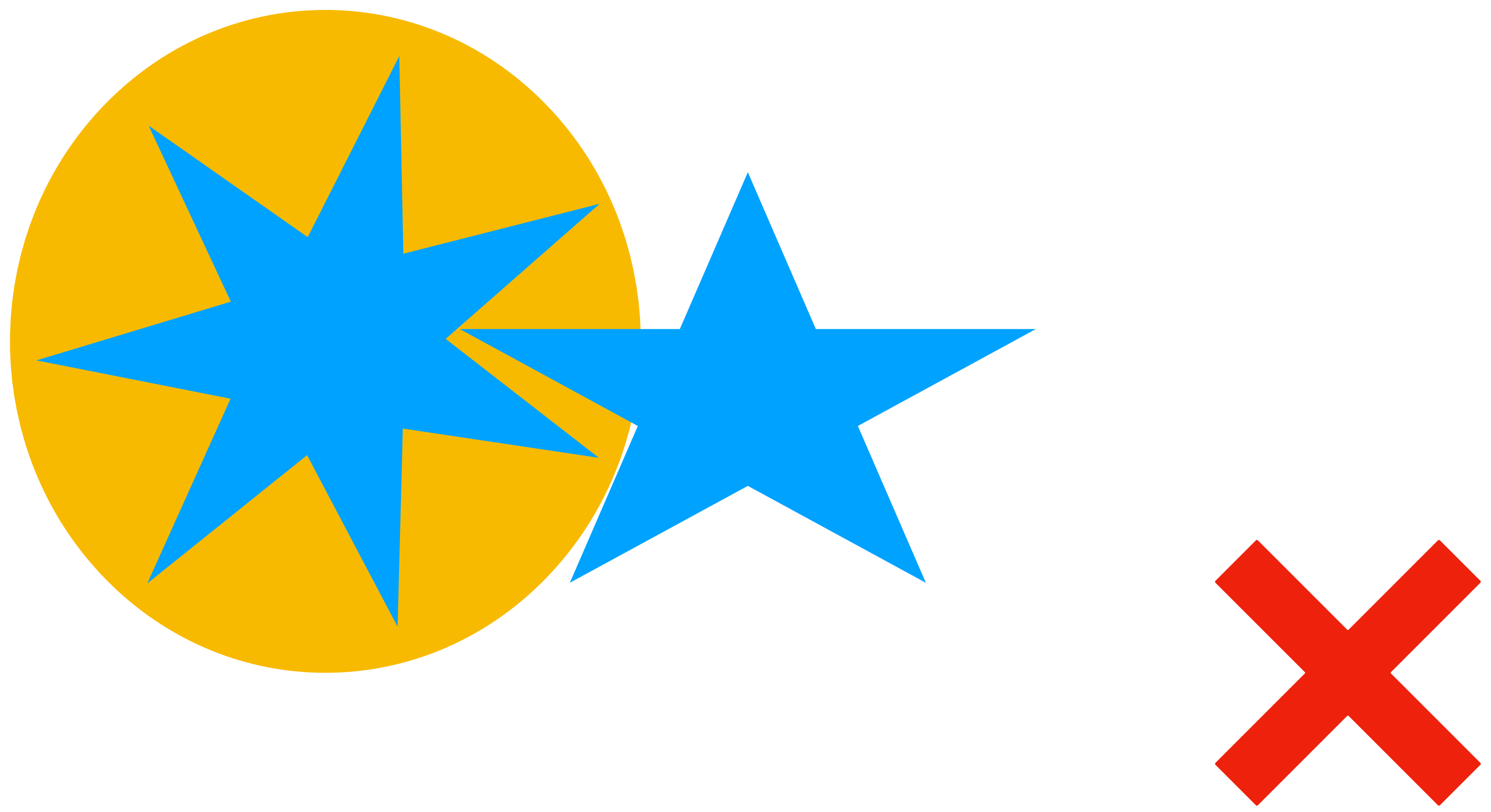

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Separability tries to understand the gap between languages.

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Insight:
Modulo seems to play an important role!

Regular Separability

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Theorem [Lorenzo, Wojtek, Slawek, Charles, ICALP'17]:
$\mathbb{Z}$-REGSEP is decidable.

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Theorem [LICS'24]:
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VASS Reachability

Deciding Reachability

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Solution:
Combine the two.

Deciding Reachability

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Coverability graphs need pumping to guarantee non-negativity.
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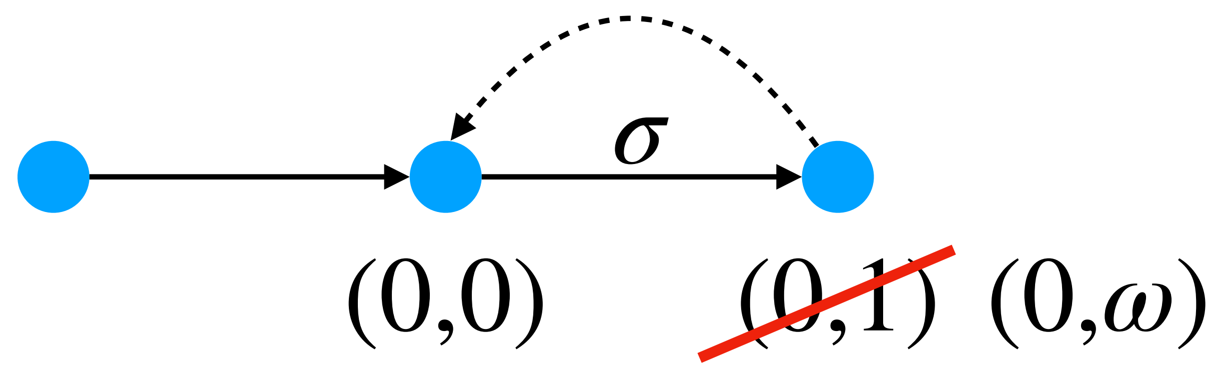

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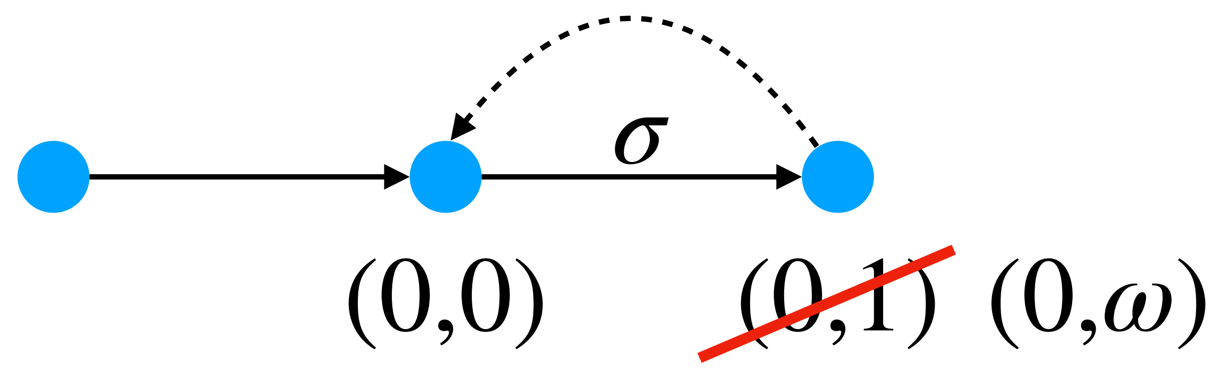

$\Rightarrow$

$x[e]$ with $e \in \sigma$
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Note: Homogeneous solutions are stable under addition.

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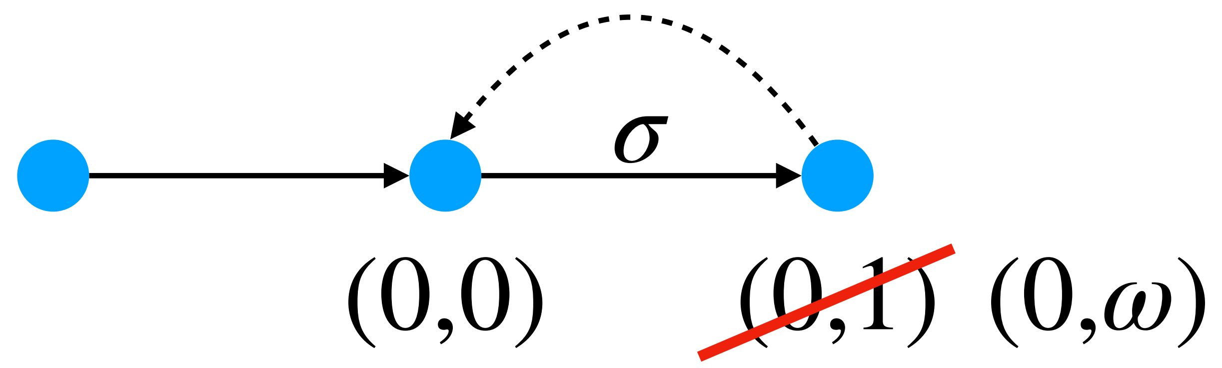

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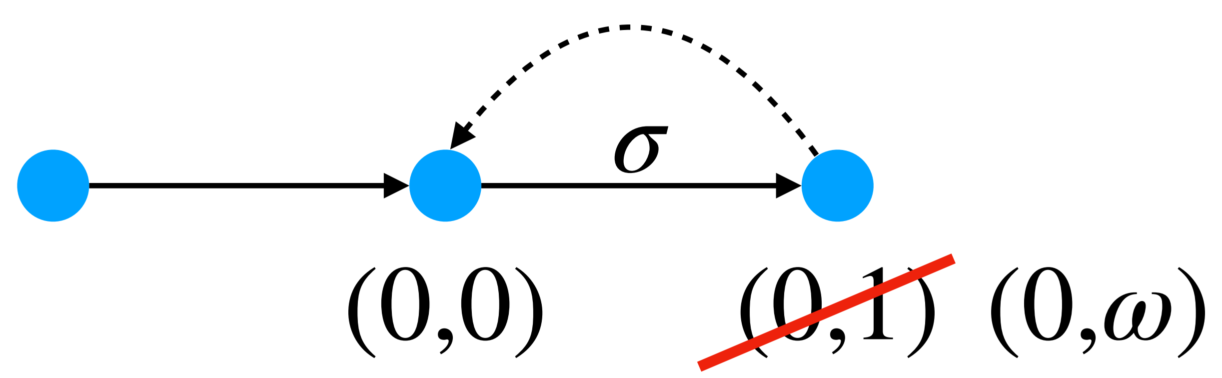
Deciding Reachability



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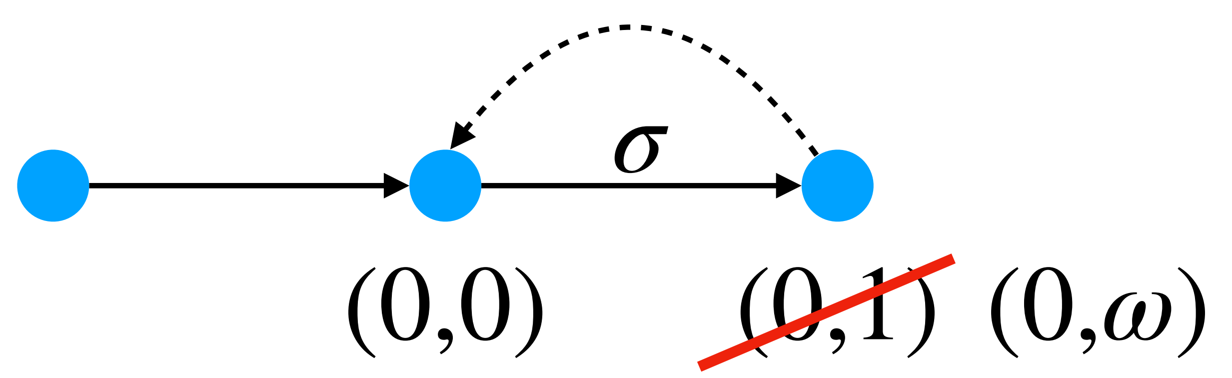

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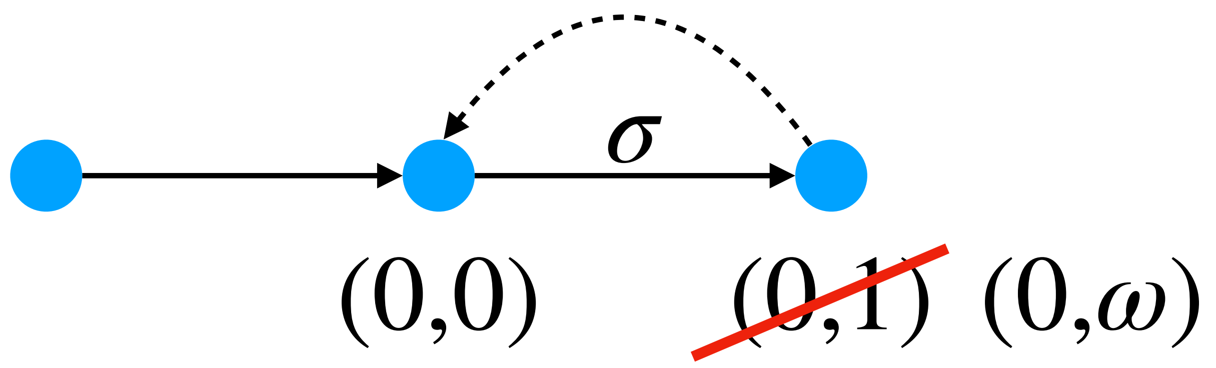

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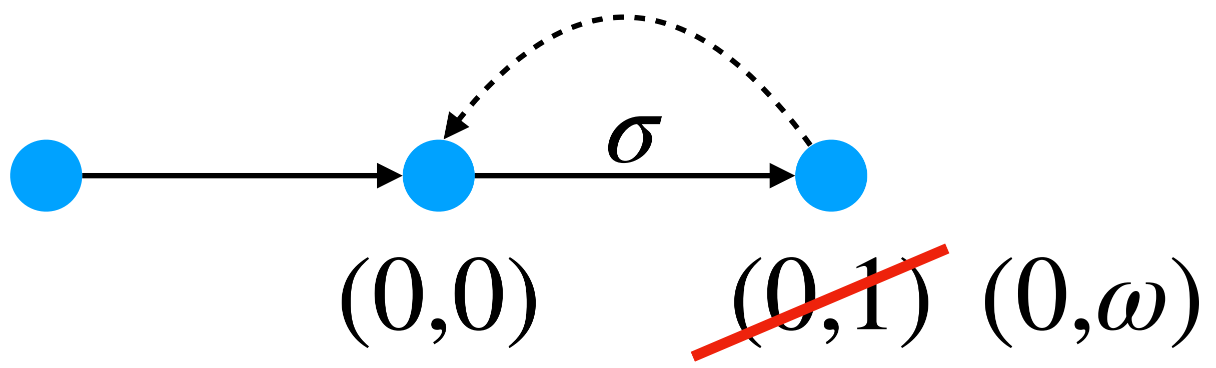

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Parikh image.

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Now $\psi(up) + \psi(w) + \psi(dn) = s$ and we say they match.

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Since pumping happens in a support solution, this still solves reachability.
Notably, it stays non-negative.

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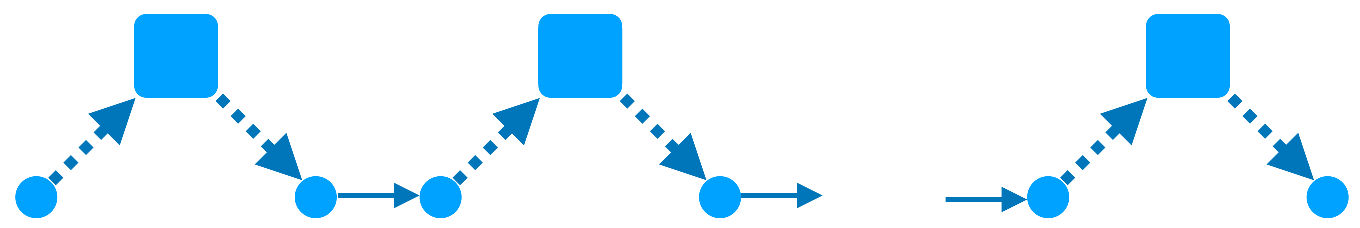

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For perfect MGTS,

$$\mathbb{N}\text{-reachability holds} \quad \Leftrightarrow \quad \mathbb{Z}\text{-reachability holds.}$$

We are hiring!

Associate Professorship in Verification (tenured, W2)

Please inform your postdocs and colleagues who may be interested!
Please contact me for questions!