



# Roaming the proof-test border



**Bertrand Meyer, Li Huang**

**Constructor Institute, Schaffhausen (CH)**

**(Work with Manuel Oriol and Ilgiz Mustafin)**

*WG2.3, Trento, 12 October 2023*

# Overall idea

Using a modern  
SMT-based  
program prover  
to derive



## **counter-examples**

for both correct and incorrect programs, hence:

- (1) **Failing tests**
- (2) **Better counter-examples**
- (3) **Full-coverage test suites**
- (4) **Automatically generated program fixes**



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# Part 1 (presented by Bertrand Meyer)

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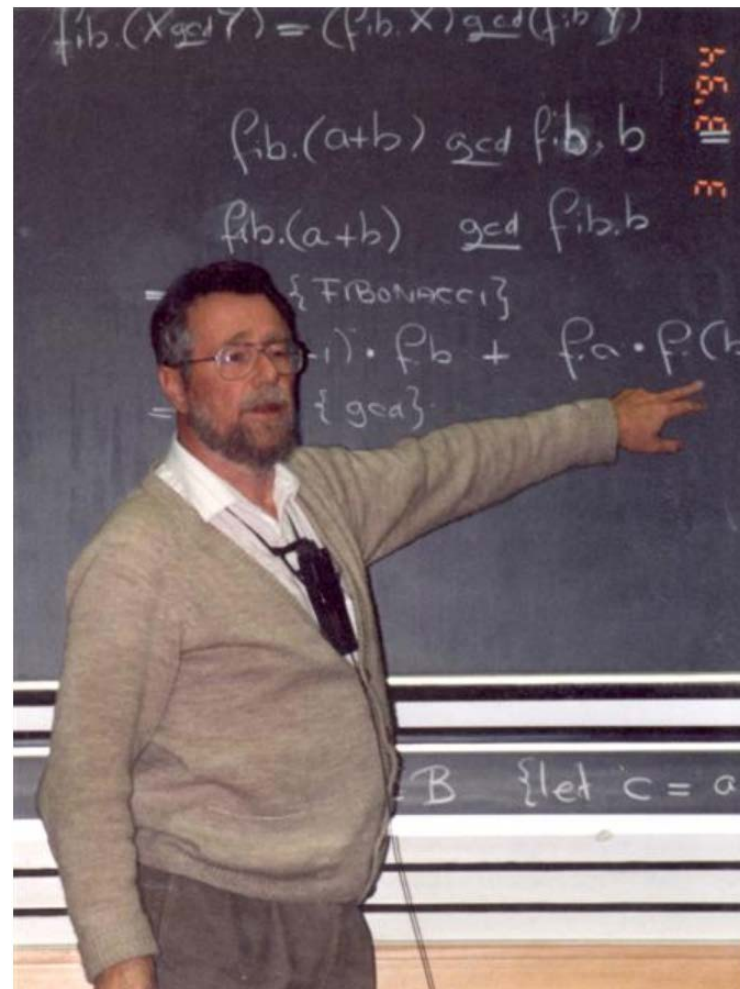
# Tests and proofs?



*The Battle of San Romano (1432) by Paolo Uccello*



Program testing  
can be used to  
show the presence  
of bugs, but never  
to show their  
absence!



# A key role of tests: the regression test suite

Consider a correct program

We shouldn't need to test it any more

But: we do want a test suite for future evolution, to spot possible **regressions**



# Tests and proofs?



*The Battle of San Romano (1432) by Paolo Uccello*





Rubens: Allegory of the Blessings of Peace (of Westphalia)

# Tests and proofs: duality



Proof

Test

Success




Failure



# Counterexamples



Some modern provers use an SMT solver:

- Attempt to prove program correct by trying to find a counterexample
  - Normally, we hope to find none, and then declare victory
- 
- A photograph of a baby laughing heartily, with its mouth wide open and eyes squinted. The baby's head and shoulders are outlined in white, making it stand out against the white background of the slide.
- If the proof attempt **fails**, it yields a counterexample
  - This counterexample is a **test** for the corresponding path

# AutoProof technology stack



Eiffel classes with  
contracts

MML



*Bernd Schoeller*

AutoProof

Boogie (prover)



*Rustan Leino*

Z3 (SMT solver)



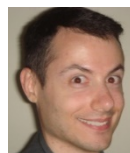
*Leonardo de Moura*



*Nikolaj Bjørner*



*Nadia Polikarpova*



*Carlo Furia*



*Julian Tschannen*



# <http://autoproof.sit.org>



## AutoProof



AutoProof is a verifier of object-oriented programs that uses [Boogie](#) as a back-end. AutoProof is an ongoing development of the Chair of Software Engineering at SIT, based on an earlier implementation at ETH Zurich.

### AutoProof online

You can use AutoProof [in your browser](#) without downloading anything. This version is limited to single-class projects.

### AutoProof as a GitHub action

You can make [AutoProof action](#) part of your GitHub project continuous development pipeline.

### Docker image with AutoProof

You can pull a [Docker image](#) with a full-fledged Linux-based distribution of AutoProof.

### Gallery of verified programs

A [software repository](#) collects a suite of benchmark problems implemented in Eiffel and verified with AutoProof. You can run verification online and see the results!

### Documentation

- [Tutorial](#): the tutorial gets you started with AutoProof.
- [Manual](#): the manual offers a more systematic description of AutoProof.

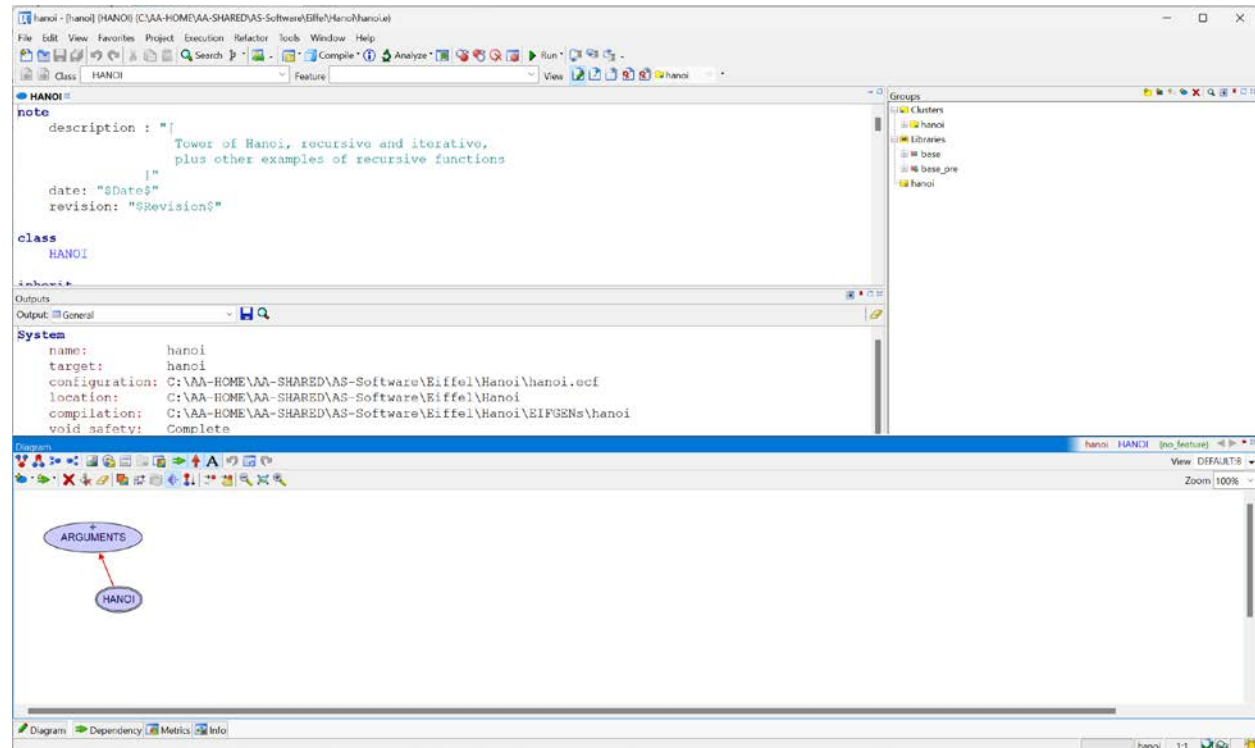
### Current contributors

# Reminder: Eiffel technology

Software development approach based on methodology, language and tools  
Encompasses entire lifecycle  
Built around principles:

Design by Contract™, Open-Closed, Command-Query Separation, Single-Choice...

Full and uncompromising application of object technology  
Supporting environment: EiffelStudio – open-source and commercial versions



# A failed proof



Proof

Test

Success

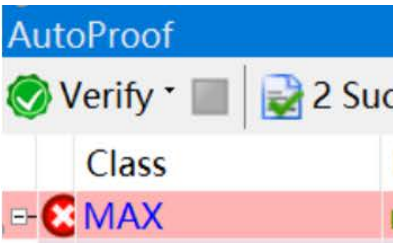


Failure

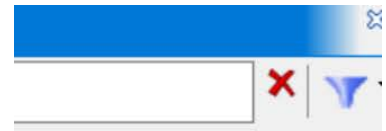
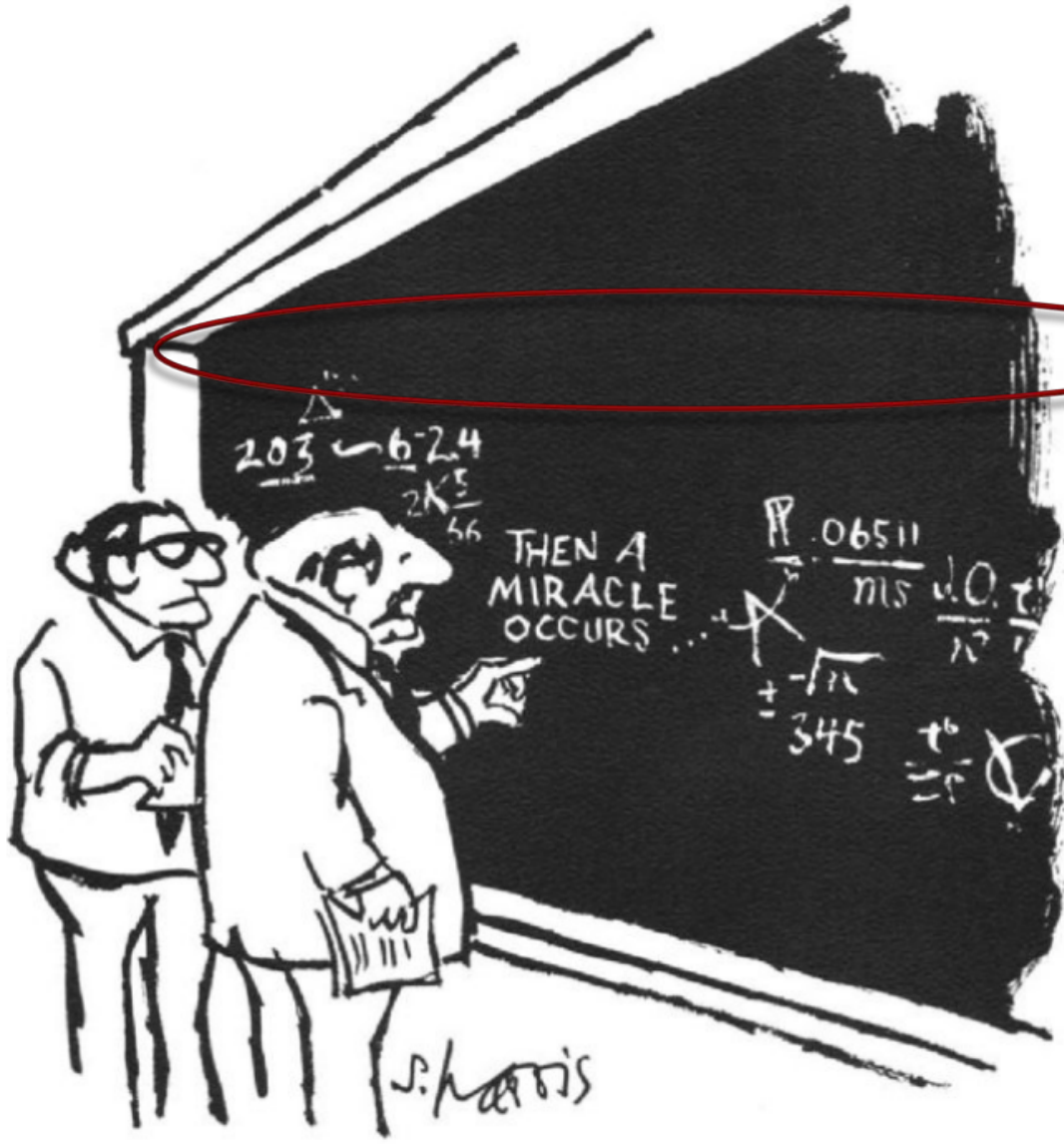


# A failed proof

yer, Oriol,  
3-



```
max (a: ARRAY
  require a.c
  local i: INT
  do
    from Res
      2 ≤ i a
      ∀ : 1 |
      ∃ j: 1 |
  until i = a
    if a
  end
  i := i + 1
  variant a
end
```



a [j] <= Result  
a [j] = Result

"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."



# Part 2 (presented by Li Huang)

---



# Counterexample generation



```
max (a: SIMPLE_ARRAY [INTEGER]): INTEGER
  require
    array_not_empty: a.count > 0
  local
    i: INTEGER
  do
    Result := a [1]
  from
    i := 2
  invariant
    i_in_bounds: 2 <= i ^ i <= a.count + 1
    max_so_far:  $\forall c: 1 \dots (i - 1) \mid a.\text{sequence}[c] \leq \text{Result}$ 
    result_in_array:  $\exists c: 1 \dots (i - 1) \mid a.\text{sequence}[c] = \text{Result}$ 
  until
    i >= a.count
  loop
    if a [i] > Result then
      Result := a [i]
    end
    i := i + 1
  variant
    a.count - i + 1
  end
ensure
  is_max:  $\forall c: 1 \dots a.\text{count} \mid a.\text{sequence}[c] \leq \text{Result}$ 
  in_array:  $\exists c: 1 \dots a.\text{count} \mid a.\text{sequence}[c] = \text{Result}$ 
end
```

AutoProof

Verify  2 Successful 1 Failed 0 Errors Filter:

	Class	Feature	Information
<input checked="" type="checkbox"/>	MAX_IN_ARRAY...	max	Postcondition <code>is_max</code> may be violated.
<input type="checkbox"/>			Counterexample: a.count = 30615, a [1] = 0, a [30614] = 0, a [30615] = 10451.

# The decipherment of an SMT model

---

```
a -> T@U!val!18  
Heap -> T@U!val!26
```

```
SIMPLE_ARRAY^INTEGER_32^.sequence -> T@U!val!9
```

```
MapType0Select -> {  
  T@U!val!26 T@U!val!18 T@U!val!9 -> T@U!val!40  
}
```

```
Seq#Item -> {  
  T@U!val!40 1 -> 0  
  T@U!val!40 30614 -> 0  
  T@U!val!40 30615 -> 10451  
}
```

```
Seq#Length -> {  
  T@U!val!40 -> 30615  
}
```

# The decipherment of an SMT model

---

```
a -> T@U!val!18 ←  
Heap -> T@U!val!26
```

```
SIMPLE_ARRAY^INTEGER_32^.sequence -> T@U!val!9
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```
MapType0Select -> {  
  T@U!val!26 T@U!val!18 T@U!val!9 -> T@U!val!40  
}
```

```
Seq#Item -> {  
  T@U!val!40 1 -> 0  
  T@U!val!40 30614 -> 0  
  T@U!val!40 30615 -> 10451  
}
```

```
Seq#Length -> {  
  T@U!val!40 -> 30615  
}
```

# The decipherment of an SMT model

---

```
a -> T@U!val!18  
Heap -> T@U!val!26 ←
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SIMPLE_ARRAY^INTEGER_32^.sequence -> T@U!val!9
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Seq#Item -> {  
  T@U!val!40 1 -> 0  
  T@U!val!40 30614 -> 0  
  T@U!val!40 30615 -> 10451  
}
```

```
Seq#Length -> {  
  T@U!val!40 -> 30615  
}
```

# The decipherment of an SMT model

---

```
a -> T@U!val!18  
Heap -> T@U!val!26
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SIMPLE_ARRAY^INTEGER_32^.sequence -> T@U!val!9 ←
```

```
MapType0Select -> {  
  T@U!val!26 T@U!val!18 T@U!val!9 -> T@U!val!40  
}
```

```
Seq#Item -> {  
  T@U!val!40 1 -> 0  
  T@U!val!40 30614 -> 0  
  T@U!val!40 30615 -> 10451  
}
```

```
Seq#Length -> {  
  T@U!val!40 -> 30615  
}
```

# The decipherment of an SMT model

```
a -> T@U!val!18
Heap -> T@U!val!26

SIMPLE_ARRAY^INTEGER_32^.sequence -> T@U!val!9

MapType0Select -> {
  T@U!val!26 T@U!val!18 T@U!val!9 -> T@U!val!40
}
```



```
a.sequence -> T@U!val!40
```

```
Seq#Item -> {
  T@U!val!40 1 -> 0
  T@U!val!40 30614 -> 0
  T@U!val!40 30615 -> 10451
}
```

```
Seq#Length -> {
  T@U!val!40 -> 30615
}
```

# The decipherment of an SMT model

```
a -> T@U!val!18
Heap -> T@U!val!26

SIMPLE_ARRAY^INTEGER_32^.sequence -> T@U!val!9

MapType0Select -> {
  T@U!val!26 T@U!val!18 T@U!val!9 -> T@U!val!40
}
```



```
a.sequence -> T@U!val!40
```

```
Seq#Item -> {
  T@U!val!40 1 -> 0
  T@U!val!40 30614 -> 0
  T@U!val!40 30615 -> 10451
}

Seq#Length -> {
  T@U!val!40 -> 30615
}
```

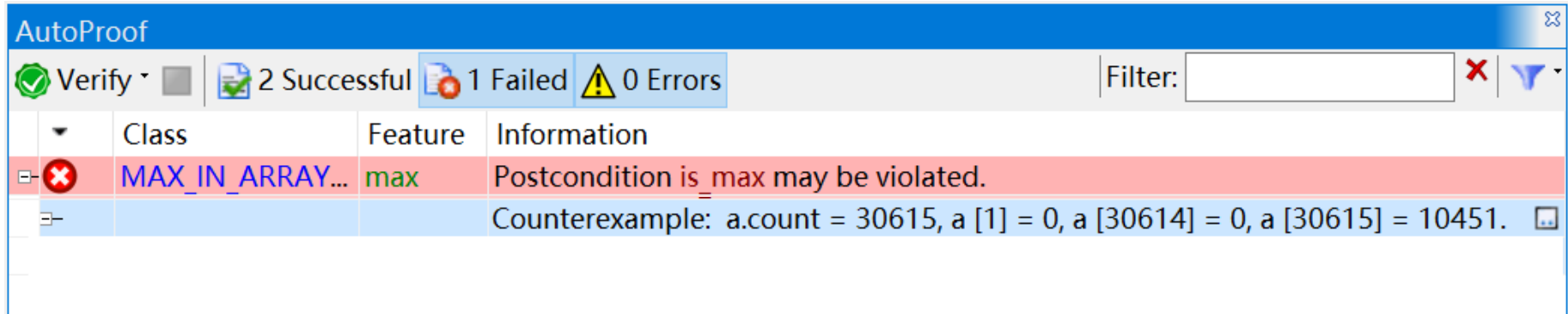


```
a [1] = 0, a [30614] = 0, a [30615] = 10451
```



# Counterexample minimization

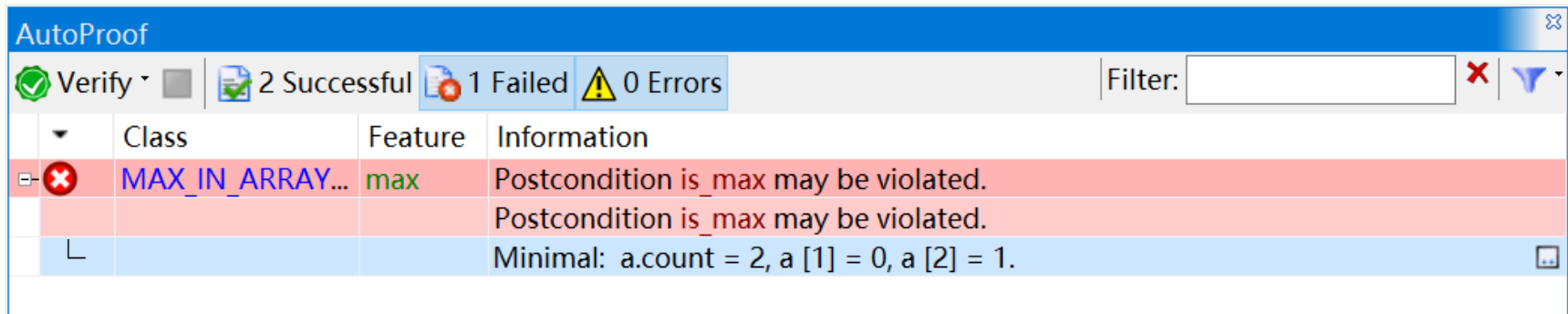
- Make the counterexample more intuitive through minimization



AutoProof

Verify  2 Successful 1 Failed 0 Errors Filter:

Class	Feature	Information
MAX_IN_ARRAY...	max	Postcondition <code>is_max</code> may be violated.
		Counterexample: <code>a.count = 30615, a [1] = 0, a [30614] = 0, a [30615] = 10451.</code>



AutoProof

Verify  2 Successful 1 Failed 0 Errors Filter:

Class	Feature	Information
MAX_IN_ARRAY...	max	Postcondition <code>is_max</code> may be violated.
		Postcondition <code>is_max</code> may be violated.
L		Minimal: <code>a.count = 2, a [1] = 0, a [2] = 1.</code>

# Counterexample minimization

Minimize each integer in the counterexample

**from**

$m \leftarrow$  current value of  $x$   
 $B.add\_precondition (0 \leq x \wedge x < m)$   
verify

**until**

No smaller value yields the same verification result

**loop**

$B.remove\_last\_precondition$   
 $m \leftarrow$  pick a smaller value  
 $B.add\_precondition (0 \leq x \wedge x < m)$   
verify

Ask prover whether it's possible to get a value of  $x$  ( $0 \leq x < m$ ) and still yields the same verification results.

When the algorithm ends (no smaller value of  $m$  can be found), the counterexample from the last verification run is the minimal possible.

# Experiment result

---

Example	Number of versions	Total Number of Minimized Integers	Avg. Reduction Rate	Avg. Number of Iterations	Avg. Verification Time (seconds)	Avg. Minimization Time (seconds)
ACCOUNT	7	17	99.98%	2.5	0.028	0.087
CLOCK	6	13	100%	1.46	0.019	0.034
HEATER	2	4	48.4%	4.25	0.030	0.128
LAMP	4	8	0.819%	1.875	0.115	0.233
BINARY_SEARCH	5	31	98.8%	3.22	0.448	1.512
LINEAR_SEARCH	3	9	99.9%	3.44	0.087	0.279
SQUARE_ROOT	4	3	89.9%	4	0.133	0.505
MAX	4	12	87.1%	4.25	0.213	1.456
SUM_AND_MAX	6	11	80.7%	3.45	0.590	1.704

**125** integers are minimized in total

**108** are minimized into values  $[-2, 2]$

**58** are minimized to 0

# Generate test script from counterexample



Class	Feature	Information
ACCOUNT	transfer	Postcondition <code>withdrawal_made</code> may be violated.
		Postcondition <code>withdrawal_made</code> may be violated.
		Counterexample: <code>balance = -2147475890, credit_limit = -2147483610, amount = 7720, other = Current.</code>
		Minimal: <code>balance = 0, credit_limit = -1, amount = 1, other = Current.</code>



```
test_ACCOUNT_transfer
  local
    current_object: ACCOUNT
    amount: INTEGER
    other: ACCOUNT
  do
    create current_object.make
    {P_INTERNAL}.set_integer_32_field_ ("balance", current_object, 0)
    {P_INTERNAL}.set_integer_32_field_ ("credit_limit", current_object, (-1))
    amount := 1
    other := current_object
    current_object.transfer (amount, other)
  end
```

# Seeding contradiction for full-coverage test suite

simple (a: INTEGER)

do

if a > 0 then

~~check False end~~

x := 1 -- Instruction 1

else

~~check False end~~

x := 2 -- Instruction 2

end

if a<sup>2</sup> > a then

~~check False end~~

x := 3 -- Instruction 3

else

~~check False end~~

x := 4 -- Instruction 4

end

end

Test cases:  
a = 0, a = 1

Branches  
not covered!

# The solution: conditional seeding



`bn := non_deterministic (0 .. N)`

`do`

`if a > 0 then`

`if bn = 1 then check False end end`

`x := 1 -- Instruction 1 (block 1)`

`else`

`if bn = 2 then check False end end`

`x := 2 -- Instruction 2 (block 2)`

`end`

`if a2 > a then`

`if bn = 3 then check False end end`

`x := 3 -- Instruction 3 (block 3)`

`else`

`if bn = 4 then check False end end`

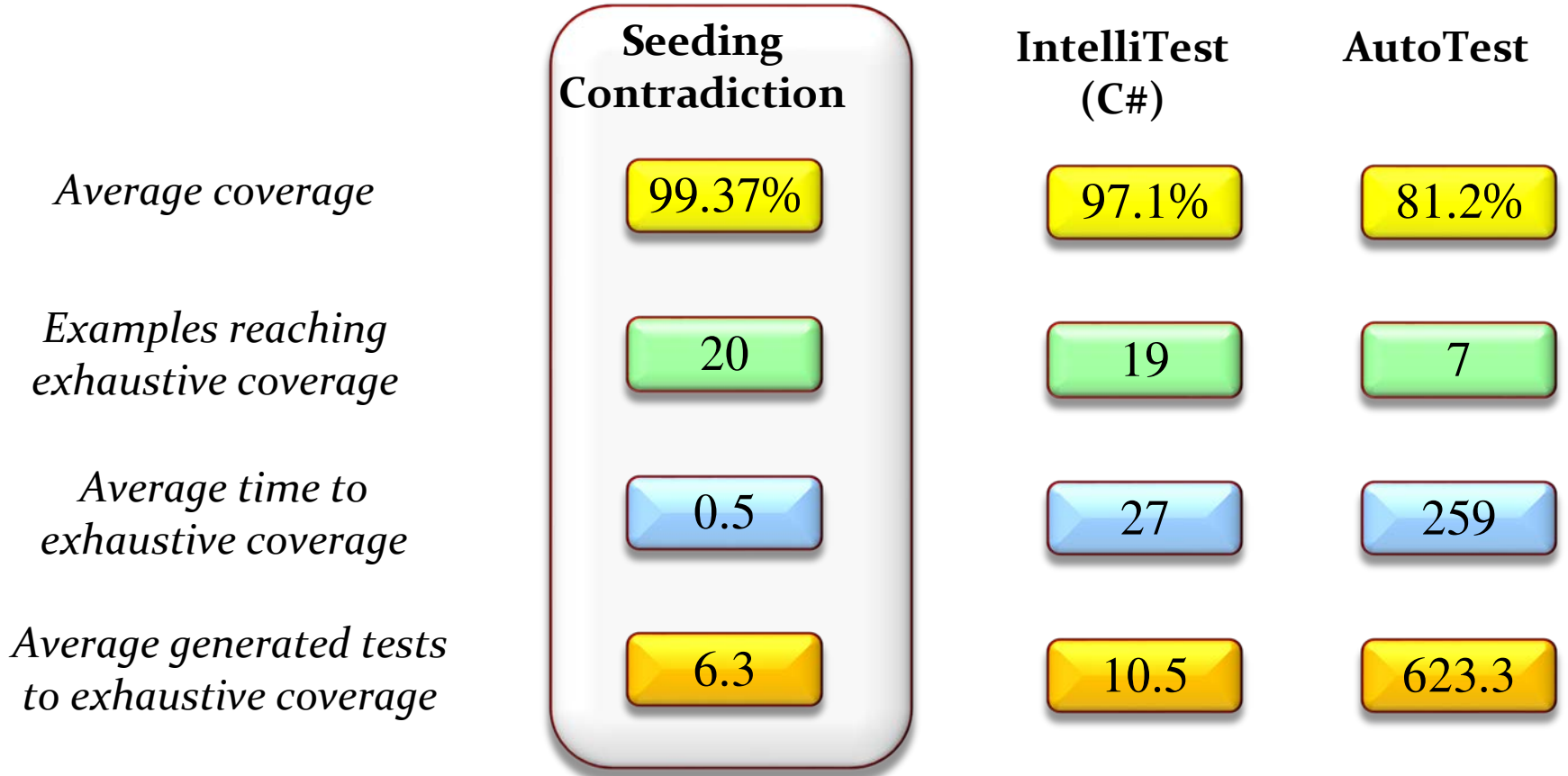
`x := 4 -- Instruction 4 (block 4)`

`end`

**N: number of  
basic blocks**

**Tests cover  
all branches!**

# Seeding contradiction: results and comparison (20 examples)



Examples (mostly from verification competitions)

	Account	Clock	Heater	Lamp	Max	Linear Search	Insertion Sort	Gnome Sort	Square root	Sum and Arithmetic max	
LOC	214	153	102	95	49	64	122	62	56	56	204
Branches	14	10	8	8	3	5	5	5	5	4	14
	Binary search	Recursive binary search	Dutch flag	Two way max	Two way sort	Two way sort	Quick sort	Selection Sort	Bubble Sort	Optimized gnome sort	Total
	74	89	188	49	85	232	167	165	183	183	2409
	5	7	11	4	6	9	5	5	8	8	141



- Limitations of SMT solver
- Some Eiffel mechanisms (genericity) not yet supported
- Single routines
- Examples still small, although some sophisticated



# The next step: generating fixes



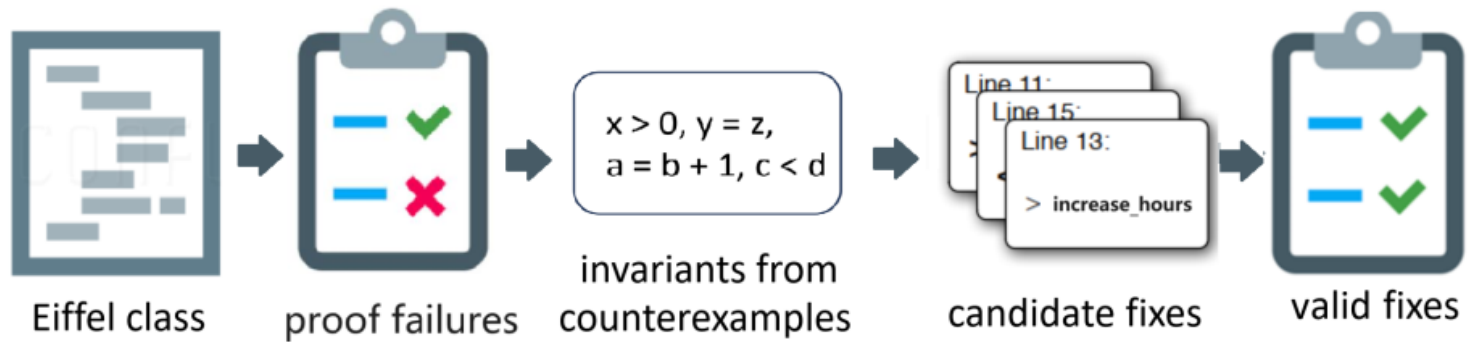
```
transfer (amount: INTEGER; other: ACCOUNT)
  -- Transfer `amount` from `Current` to `other`.
note
  explicit: wrapping
require
  other_not_void: other /= Void
  amount_not_negative: amount >= 0
  amount_not_too_large: amount <= balance
do
  withdraw (amount)
  other.deposit (amount)
ensure
  -- Allowed to modify the state of `Current` and `other`
  -- (by default a procedure can only modify `Current`):
  modify (Current, other)
  balance_decreased: balance = old balance - amount
  other_balance_increased: other.balance = old other.balance + amount
end
```

AutoProof

Verify  4 Successful  1 Failed  0 Errors

Class	Feature	Information
ACCOUNT	transfer	Postcondition <code>balance_decreased</code> may be violated.
		Postcondition <code>balance_decreased</code> may be violated.
		Counterexample: balance = 21239, amount = 1, other = Current.
		Counterexample: balance = 21239, amount = 1, other = Current.
		Counterexample: balance = 6335, amount = 1, other = Current.
		Counterexample: balance = 1, amount = 1, other = Current.
		Counterexample: balance = 9295, amount = 1, other = Current.
		Counterexample: balance = 17946, amount = 1, other = Current.
		Counterexample: balance = 12256, amount = 1, other = Current.
		Counterexample: balance = 6732, amount = 59, other = Current.
		Counterexample: balance = 15217, amount = 1, other = Current.
		Counterexample: balance = 28171, amount = 1, other = Current.
		Counterexample: balance = 6152, amount = 1, other = Current.
		Counterexample: balance = 16900, amount = 1, other = Current.
		Counterexample: balance = 21137, amount = 1, other = Current.
		Counterexample: balance = 5706, amount = 78, other = Current.
		Counterexample: balance = 1, amount = 1, other = Current.

# Proof2Fix: generating fixes for proof failures



# Example: CLOCK



```
increase_hours
```

```
do
```

```
if hours = 24 then
```

```
  hours := 0
```

```
else
```

```
  hours := hours + 1
```

```
end
```

```
end
```

```
invariant
```

```
  hours_valid:  $0 \leq \text{hours} \wedge \text{hours} \leq 23$ 
```

```
increase_minutes.
```

```
do
```

```
if minutes < 59 then
```

```
  minutes := minutes + 1
```

```
else
```

```
  minutes := 0
```

```
end
```

```
ensure
```

```
hours_increased: old minutes = 59  $\implies$  hours = (old hours + 1) \24
```

```
end
```

AutoProof				
Verify  6 Successful  2 Failed  0 Errors				
	Feature	Information	Position	Ti...
	increase_hours	Invariant hours_valid might not hold.	8	0.02
	increase_minutes	Postcondition hours_increased may be violated.	16	0.01

# Example: CLOCK



```
increase_hours_fixed
do
  if hours = 23 then
    hours := 0
  else
    if hours = 24 then
      hours := 0
    else
      hours := hours + 1
    end
  end
end
end
```

```
increase_minutes_fixed
do
  if minutes = 59 then
    increase_hours
  end
  if minutes < 59 then
    minutes := minutes + 1
  else
    minutes := 0
  end
end
```

The screenshot shows the AutoProof window with a blue title bar. Below the title bar is a status bar with icons for 'Verify' (checked), '8 Successful', '0 Failed', and '0 Errors'. A 'Filter:' input field is on the right. Below the status bar is a table with the following data:

	Feature	Information	Position	Ti...
✓	increase_hours	Verification successful.		0.05
✓	increase_minutes	Verification successful.		0.01
✓	increase_seconds	Verification successful.		0.00



# Generating candidate fixes

Candidate fixes based on a counterexample invariant  $\phi$

## Fixes on contracts

- **Precondition strengthening:** add **not**  $\phi$  to  $r$ 's precondition, to rule out the faulty cases characterized by  $\phi$ .
- **Postcondition weakening:** if  $\psi$  is the postcondition clause that causes the proof to fail, replace it by **not**  $\phi$  **implies**  $\psi$ , so that the previously failing cases will now verify.



# Generating candidate fixes

Candidate fixes based on a counterexample invariant  $\phi$

## Fixes on implementation

Replace the implementation with the code snippet generated based on the following schema:

```
if  $\phi$  then  
  snippet  
end  
old_stmt
```

```
if  $\phi$  then  
  snippet  
else  
  old_stmt  
end
```

# Fixing Results of Proof2Fix



Classes	LOC	#Fail	#Fixed	Avg.#Cand	Avg.#Valid	Avg. $T_f$ (m)
ACCOUNT	97	7	3	140	5	1.9
CLOCK	131	8	4	337	8	2.7
HEATER	73	4	4	432	21	4.5
LAMP	71	4	3	454	6	4.6
ARITHMETIC	176	3	2	26	8	1.1
BINARY_SEARCH	50	6	0	–	–	–
MAX_IN_ARRAY	33	6	0	–	–	–
SQUARE_ROOT	38	4	3	9	1	1.6
V_ARRAY	1756	1	1	267	6	2.4
V_ARRAYED_LIST	1090	1	1	121	9	9.4
V_INDEXABLE_SET	1125	1	1	281	7	2.4
V_LINKED_LIST	2445	2	2	457	15	2.3
<b>Total</b>	7085	47	24	252	8	3



## Take advantage of the test-proofs complementarity

- Generate failing tests from failing proofs
- Make these tests meaningful to programmers
- For a correct program, generate a test suite:
  - Guaranteed exhaustive coverage
  - Does not require any test data
  - Based on the program text only
  - Entirely automatic
  - Extremely fast
- Next: Automatic Program Repair with the same benefits



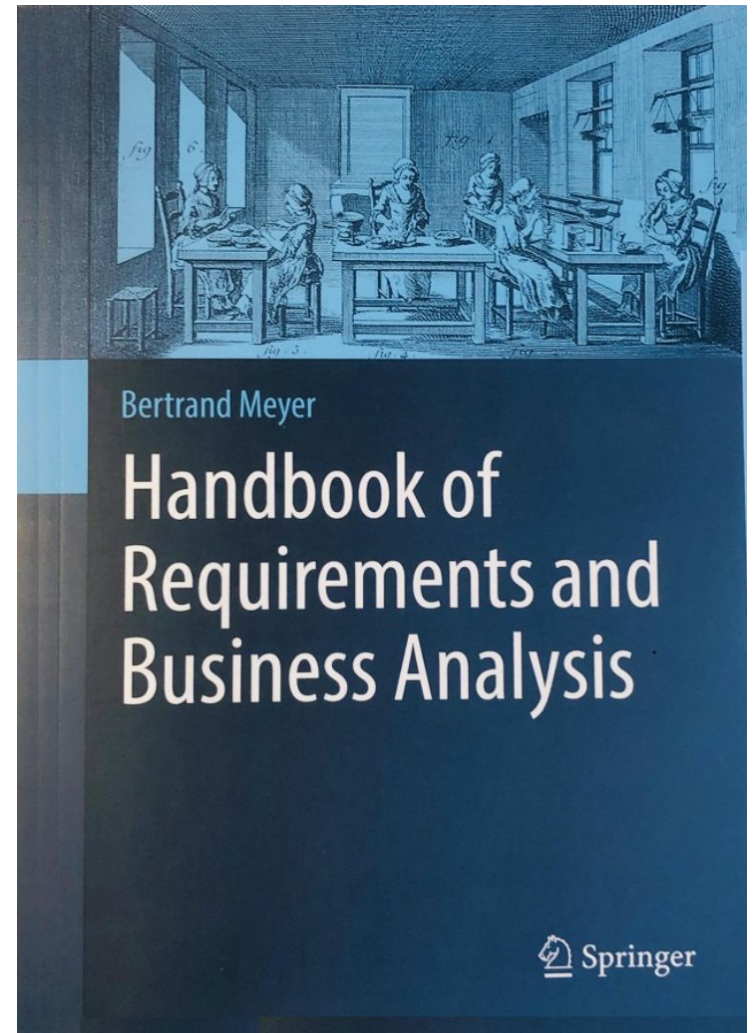
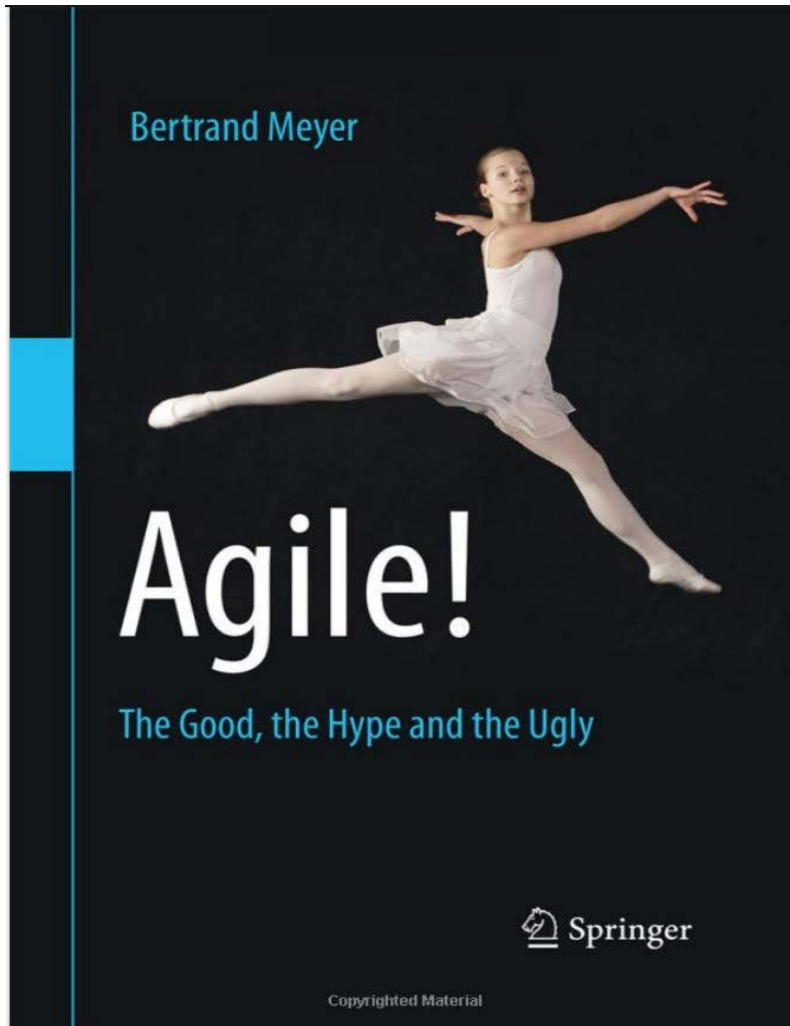


Master programs (CSSE-Leadership/Quantum)

PhD and postdoc positions in SE, quantum, verification...



# (Fairly) recent books



# For more!

AutoProof

<http://autoproof.sit.org>

Eiffel

<https://eiffel.com>    <https://eiffel.org>

Constructor Institute

<https://constructor.org/institute>

