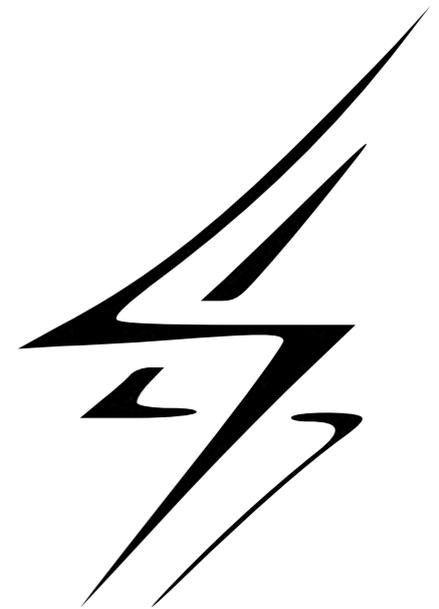


# Optimizing synthesis with metasketches

James Bornholt, Emina Torlak,  
Dan Grossman, and Luis Ceze

University of Washington



# outline

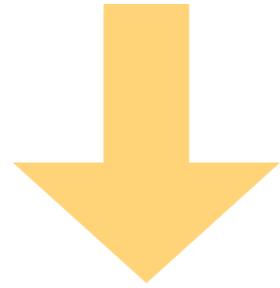
- 1. introduction**
- 2. metaskeches**
- 3. synapse** 
- 4. results**

# intro

**synthesis with sketches**

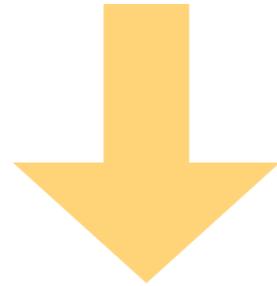
# **synthesis** with sketches

**specification**

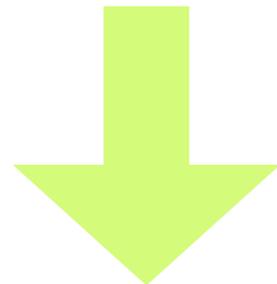


**synthesis** with sketches

**specification**

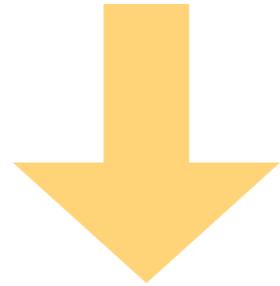


**synthesis** with sketches

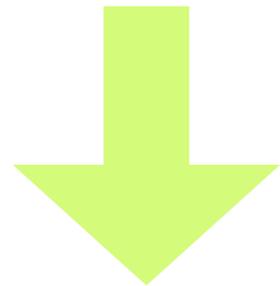


**implementation**

$$f(x) = 4 * x$$

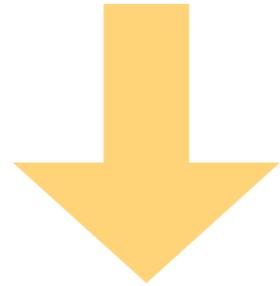


**synthesis** with sketches

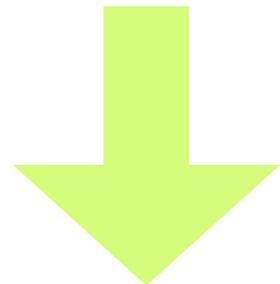


**implementation**

$$f(x) = 4 * x$$



**synthesis** with sketches



```
def f(x):  
    return x+x+x+x
```

$$f(x) = 4 * x$$

```
def f(x):  
    return Expr
```

```
Expr := x | ?? | Expr op Expr  
op := + | * | - | >> | <<  
?? := integer constant
```

**synthesis with sketches**

```
def f(x):  
    return x+x+x+x
```

$f(x) = 4 * x$

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def f(x):  
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Expr := x | ?? | Expr op Expr  
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```

**synthesis with sketches**

**Counterexample-  
Guided Inductive  
Synthesis (CEGIS)**

```
def f(x):  
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```

$$f(x) = 4 * x$$

```
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```

**synthesis with sketches**

guess, check, learn

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def f(x):  
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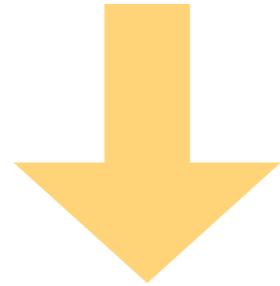
**synthesis with sketches**

guess, check, learn

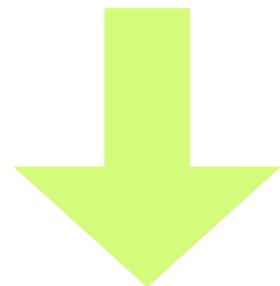
```
def f(x):  
    return x+x+x+x
```

candidate programs

$f(x) = 4 * x$



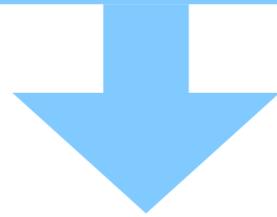
**synthesis with sketches**



```
def f(x):  
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```

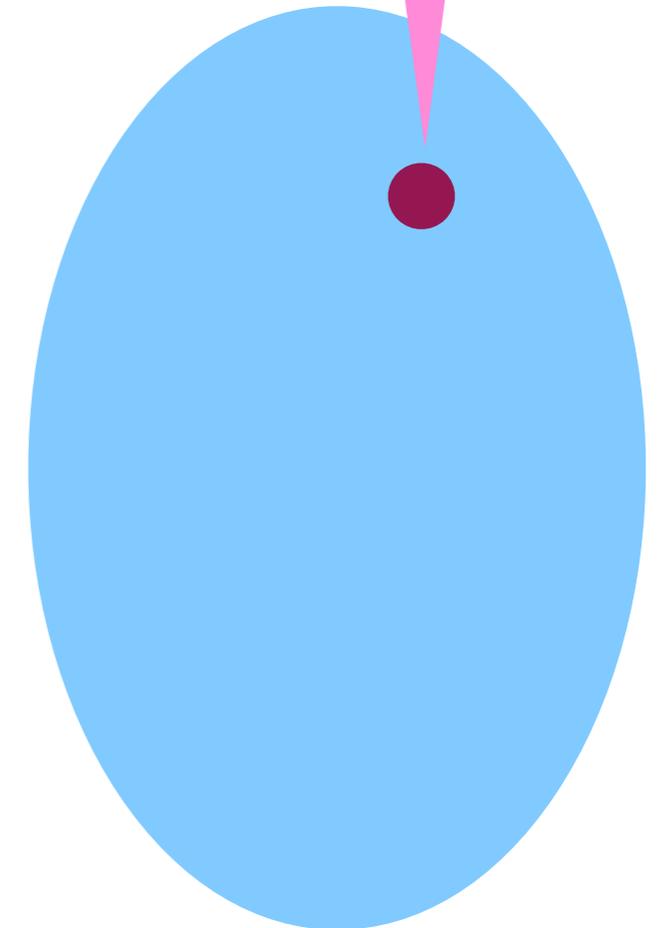
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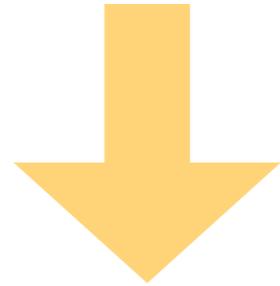
guess, check, learn

```
def f(x):  
    return x+1
```

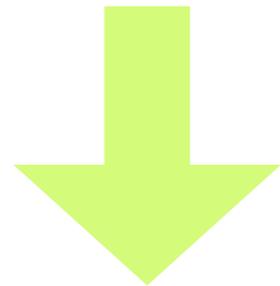


candidate programs

$$f(x) = 4 * x$$



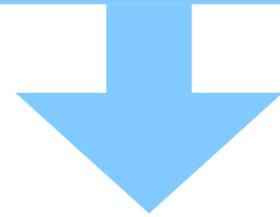
**synthesis with sketches**



```
def f(x):  
    return x+x+x+x
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def f(x):  
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Expr := x | ?? | Expr op Expr  
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```



guess, **check**, learn

```
def f(x):  
    return x+1
```

$f(0) \neq 0$



candidate programs

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**synthesis with sketches**

guess, check, **learn**

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def f(x):  
    return x+x+x+x
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```
def f(x):  
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```

$f(0) \neq 0$

**X**

candidate programs

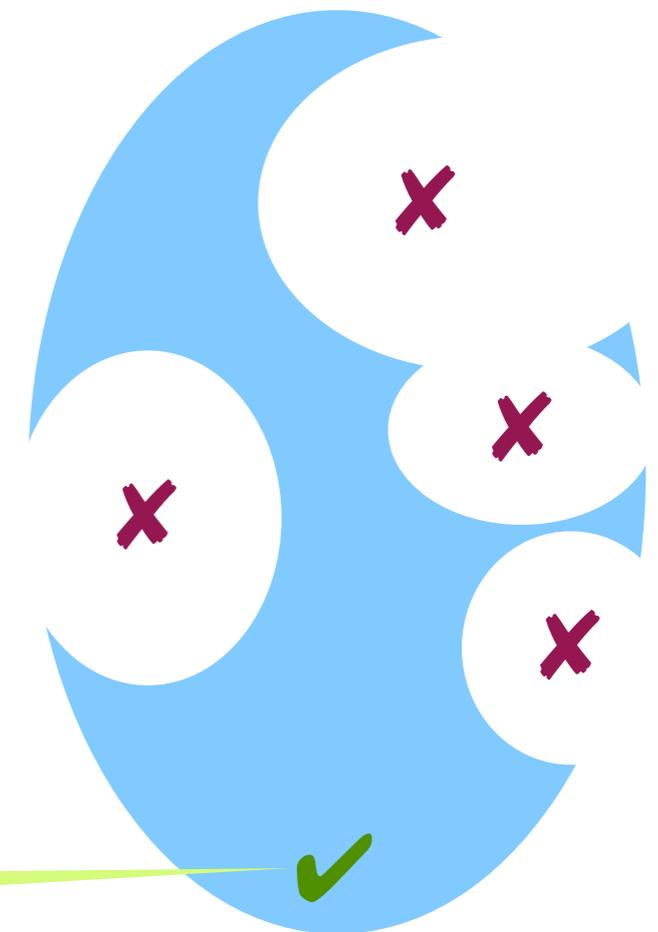
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**synthesis with sketches**

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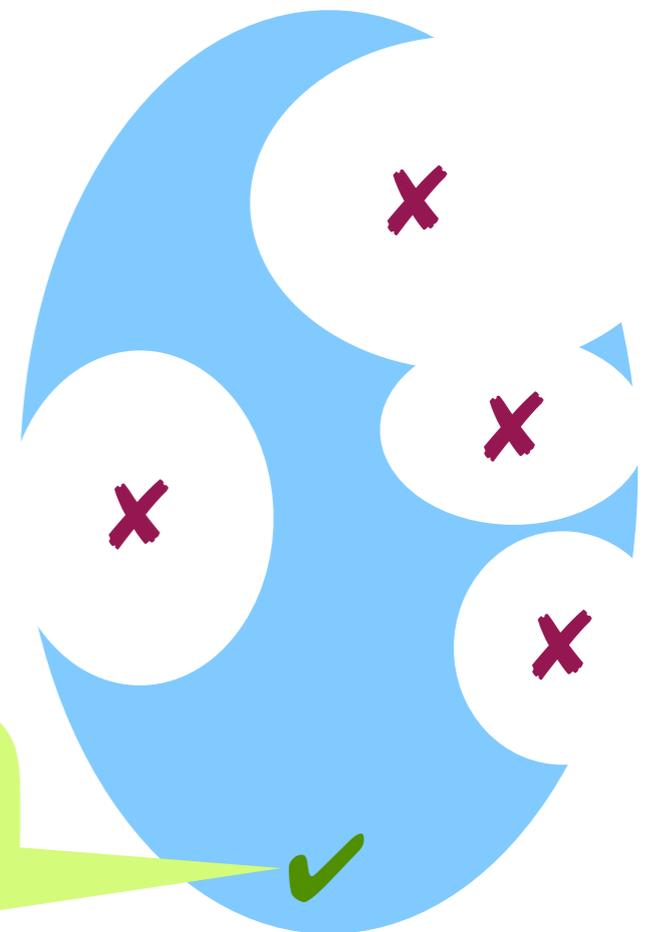


candidate programs

# building a practical synthesizer is hard ...

$$f(x) = 4 * x$$

```
def f(x):  
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```



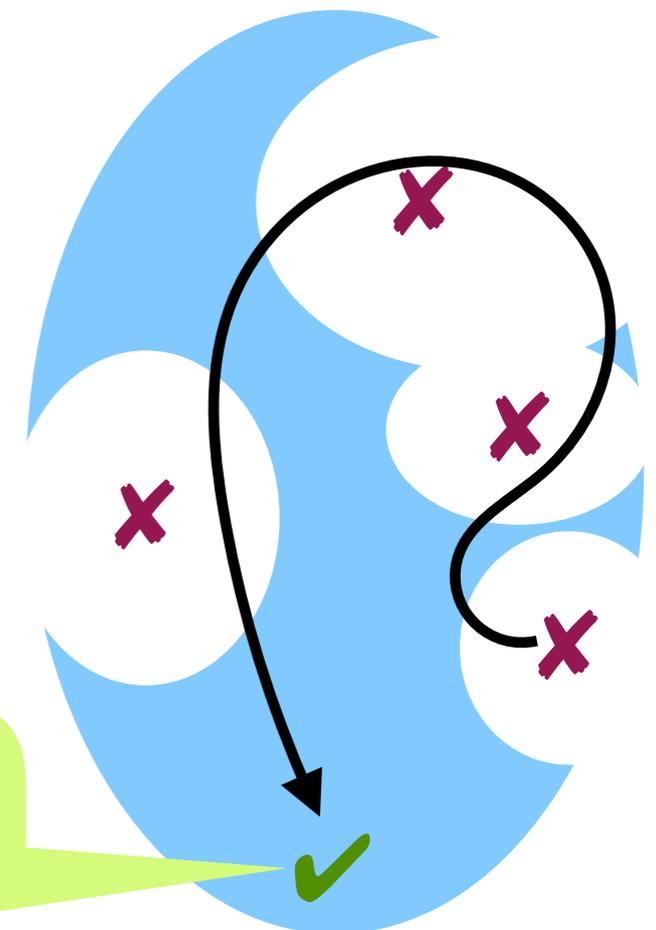
candidate programs

# building a practical synthesizer is hard ...

$$f(x) = 4 * x$$

I. pick the right search strategy

```
def f(x):  
    return x+x+x+x
```



candidate programs

# building a practical synthesizer is hard ...

1. pick the right search strategy
2. find the *best* correct program

$$f(x) = 4 * x$$

```
def f(x):  
    return x >> 2
```

candidate programs

# building a practical synthesizer is hard ...

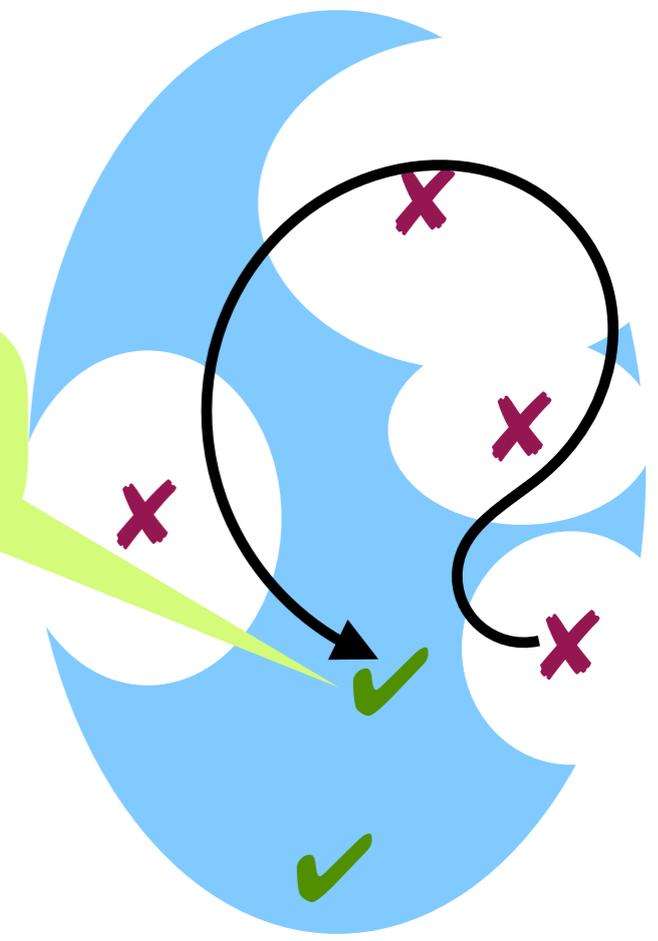
Existing tools hardcode both the search strategy and cost metric (if any), so are difficult to reuse for new domains.

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candidate programs



# building a practical synthesizer is hard ...

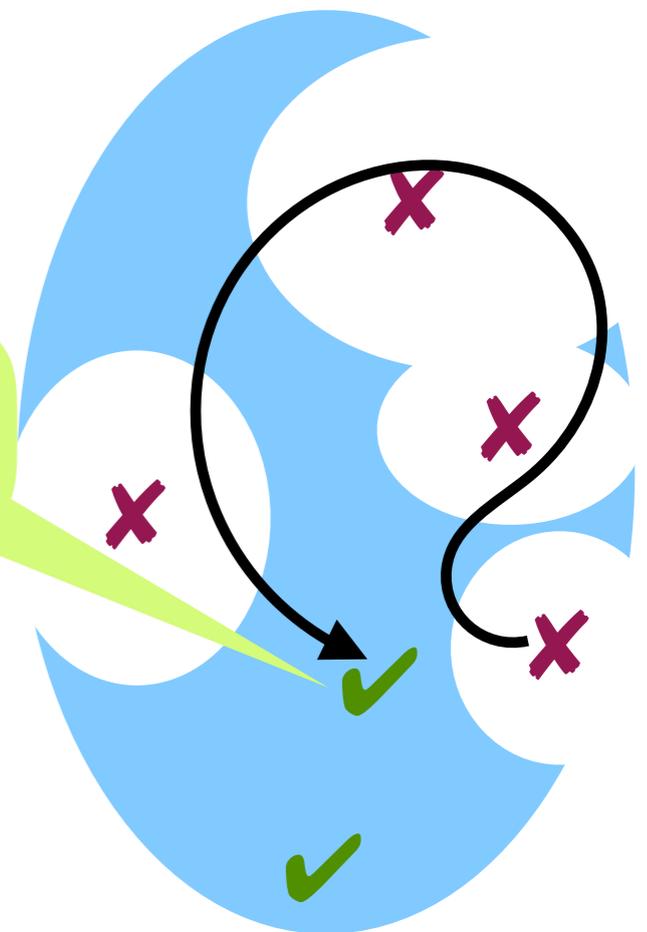
Existing tools hardcode both the search strategy and cost metric (if any), so are difficult to reuse for new domains.

1. pick the right search strategy
2. find the *best* correct program

*Metasketches* are a new way to express synthesis problems, making the search strategy and the cost function explicit in the problem definition.

$$f(x) = 4 * x$$

```
def f(x):  
    return x >> 2
```



candidate programs

design

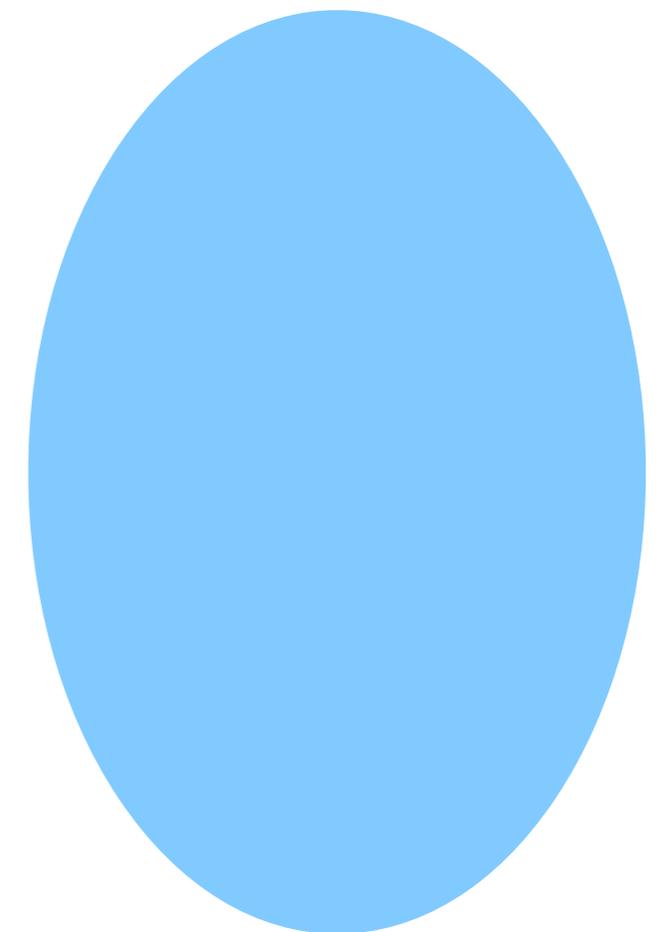
**metasketches**

# **anatomy of a metasketch**

- 1. structured candidate space ( $\mathcal{S}, \preceq$ )**
- 2. cost function ( $\kappa$ )**
- 3. gradient function ( $g$ )**

# anatomy of a metasketch

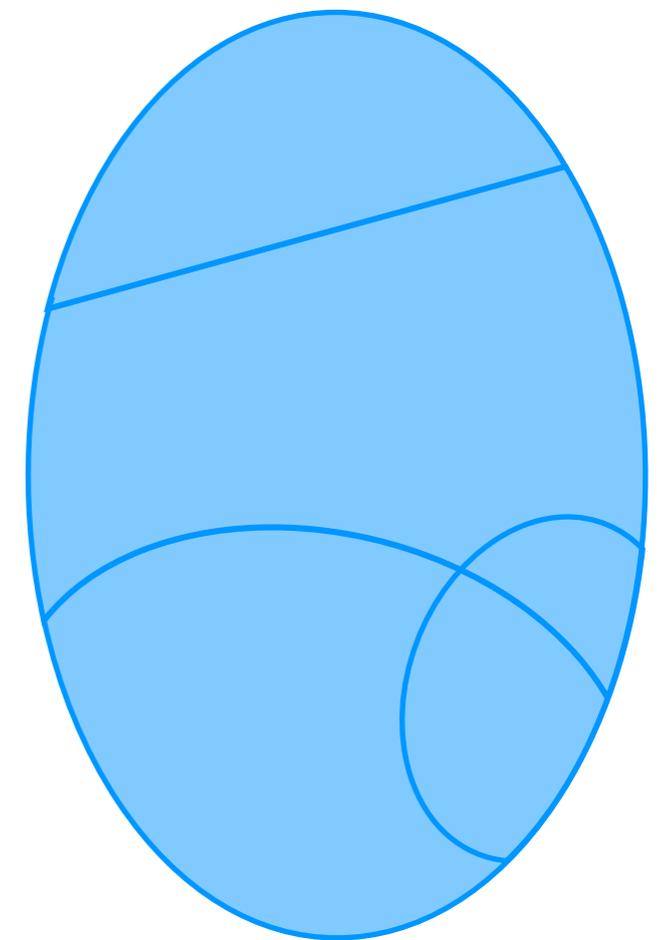
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candidate programs

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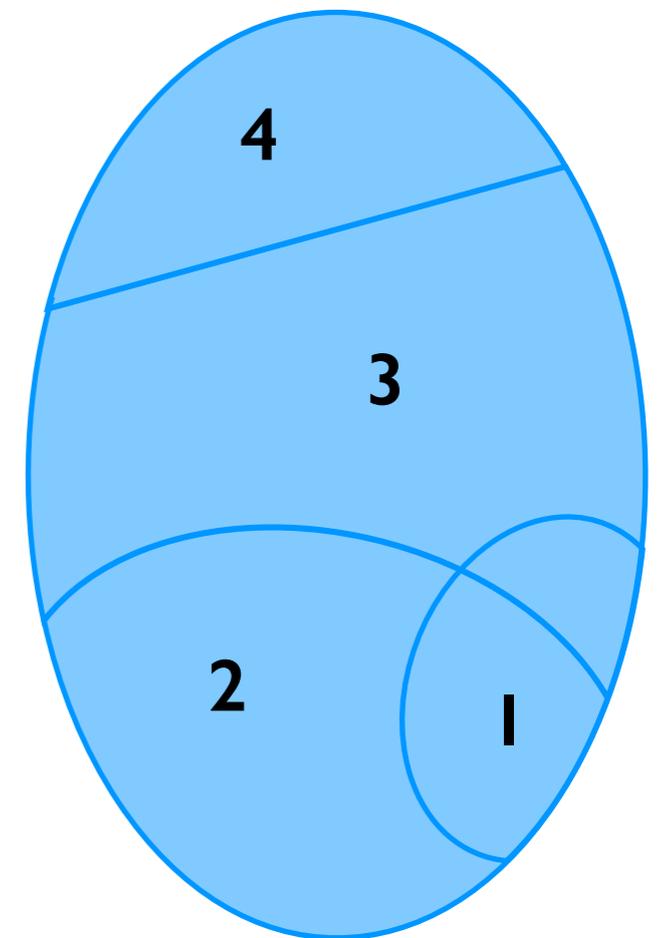
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candidate programs

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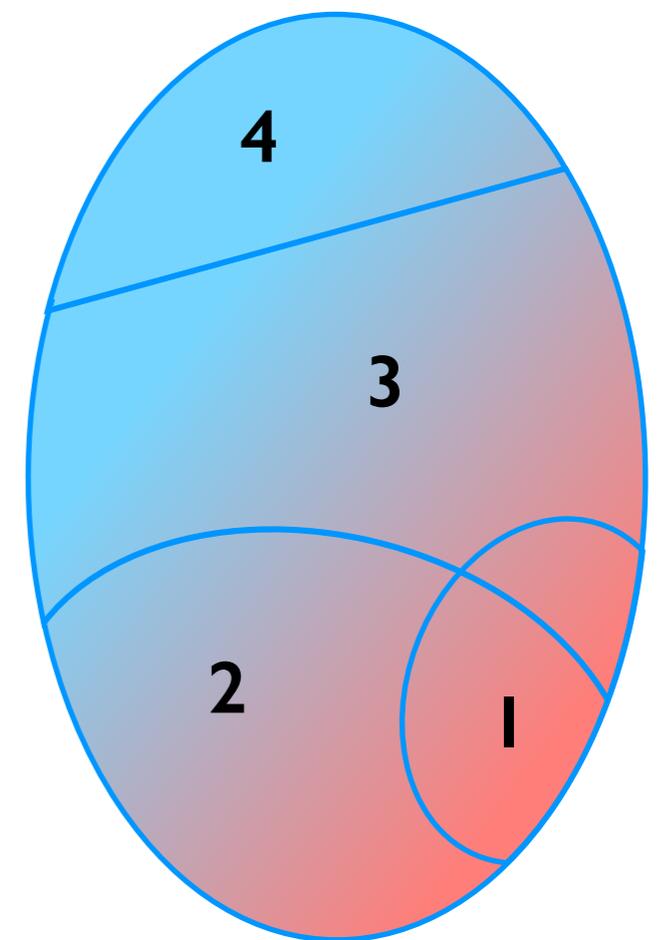
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candidate programs

# anatomy of a metasketch

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candidate programs

# anatomy of a metasketch (I)

## 1. structured candidate space ( $\mathcal{S}, \preceq$ )

- ▶ a countable set  $\mathcal{S}$  of sketches
- ▶ a total order  $\preceq$  on  $\mathcal{S}$

## 2. cost function ( $\kappa$ )

## 3. gradient function ( $g$ )

# anatomy of a metasketch (I)

## 1. structured candidate space $(\mathcal{S}, \preceq)$

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## 2. cost function $(\kappa)$

## 3. gradient function $(g)$

A set of sketches  $\mathcal{S}$  can express candidate spaces that cannot be expressed with a single sketch or a CFG.

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$\mathcal{S}$  (space of all SSA programs)

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  return r1
```

$S_1$

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  r2 = [x|r1|??] [+|...] [x|r1|??]  
  return r2
```

$S_2$

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  r2 = [x|r1|??] [+|...] [x|r1|??]  
  r3 = [x|r1|r2|??] [+|...] [x|r1|r2|??]  
  return r3
```

$S_3$



# anatomy of a metasketch (I)

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A set of sketches  $\mathcal{S}$  can express candidate spaces that cannot be expressed with a single sketch or a CFG.

The ordering  $\preceq$  on sketches expresses a high-level search strategy.

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```

$S_2$

```
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  r2 = [x|r1|??] [+|...] [x|r1|??]  
  return r2
```

$S_3$

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  r2 = [x|r1|??] [+|...] [x|r1|??]  
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  return r3
```



# anatomy of a metasketch (I)

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$\mathcal{S}, \preceq$  (SSA with iterative deepening)

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$\preceq$

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$\preceq$

$S_3$

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$\preceq$

•  
•  
•

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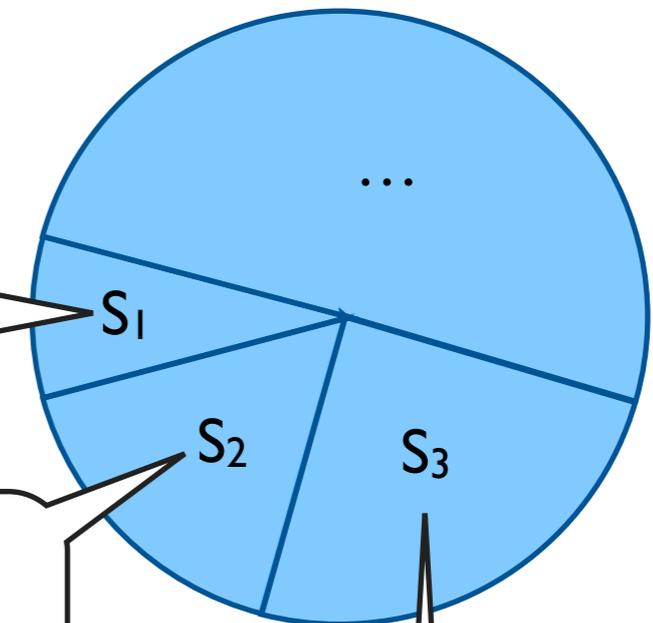
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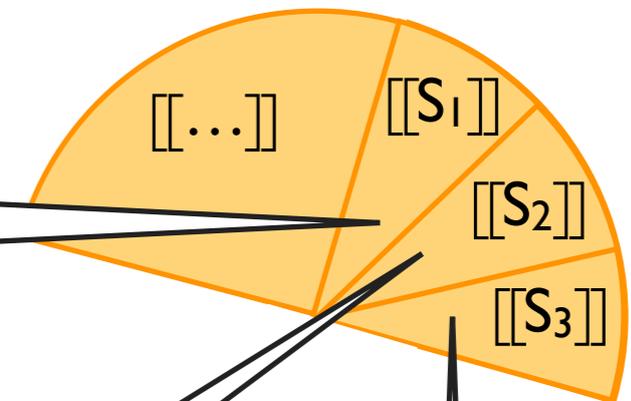
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def f(x):  
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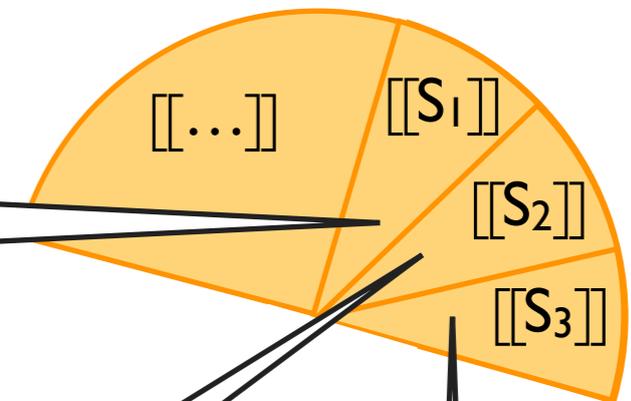
## 2. cost function ( $\kappa$ )

## 3. gradient function ( $g$ )

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  return r1
```

```
def f(x):  
  r1 = e11 [+|...] e12  
  r2 = e21 [+|...] e22  
  return r2
```

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  r2 = [x|r1|??] [+|...] [x|r1|??]  
  r3 = [x|r1|r2|??] [+|...] [x|r1|r2|??]  
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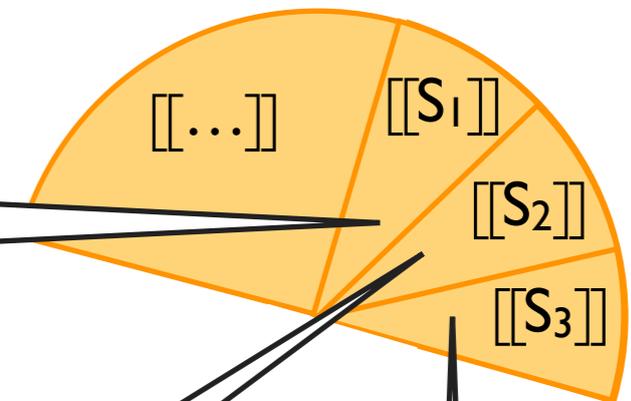
## 2. cost function ( $\kappa$ )

## 3. gradient function ( $g$ )

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  return r1
```

```
def f(x):  
  r1 = e11 [+|...] e12  
  r2 = e21 [+|...] e22  
  assert r1 == e21 || r1 == e22  
  return r2
```

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  r2 = [x|r1|??] [+|...] [x|r1|??]  
  r3 = [x|r1|r2|??] [+|...] [x|r1|r2|??]  
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# anatomy of a metasketch (I)

## 1. structured candidate space ( $\mathcal{S}, \preceq$ )

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## 2. cost function ( $\kappa$ )

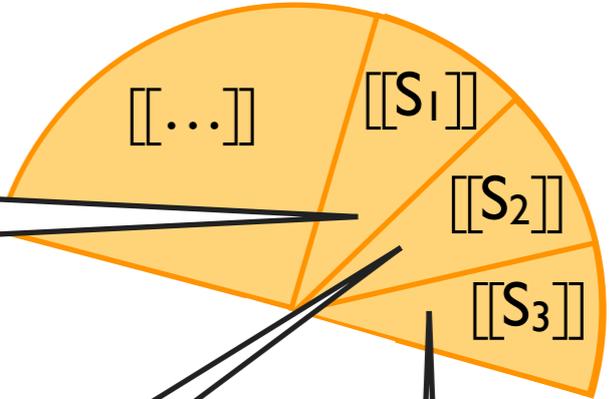
## 3. gradient function ( $g$ )

Structure constraints help reduce semantic redundancies in the search space.

```
def f(x):  
  r1 = [x|??] [+|...] [x|??]  
  return r1
```

```
def f(x):  
  r1 = e11 [+|...] e12  
  r2 = e21 [+|...] e22  
  assert r1 == e21 || r1 == e22  
  return r2
```

```
def f(x):  
  r1 = e11 [+|...] e12  
  r2 = e21 [+|...] e22  
  r3 = e31 [+|...] e32  
  assert r1 == e21 || ... || r1 == e32  
  assert r2 == e31 || r2 == e32  
  return r3
```



# anatomy of a metasketch (2)

1. structured candidate space ( $\mathcal{S}, \preceq$ )

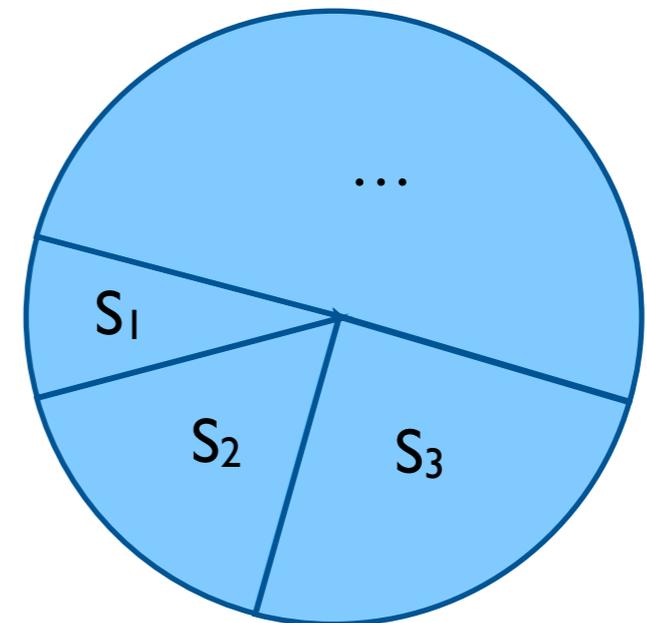
$\mathcal{S}, \preceq$  (SSA with iterative deepening)

2. cost function ( $\kappa$ )

▶  $\kappa : \{ P \mid P \in S_i \wedge S_i \in \mathcal{S} \} \rightarrow \mathbb{R}$

▶ assigns a numeric cost to each candidate

3. gradient function ( $g$ )



# anatomy of a metasketch (2)

## 1. structured candidate space ( $\mathcal{S}, \preceq$ )

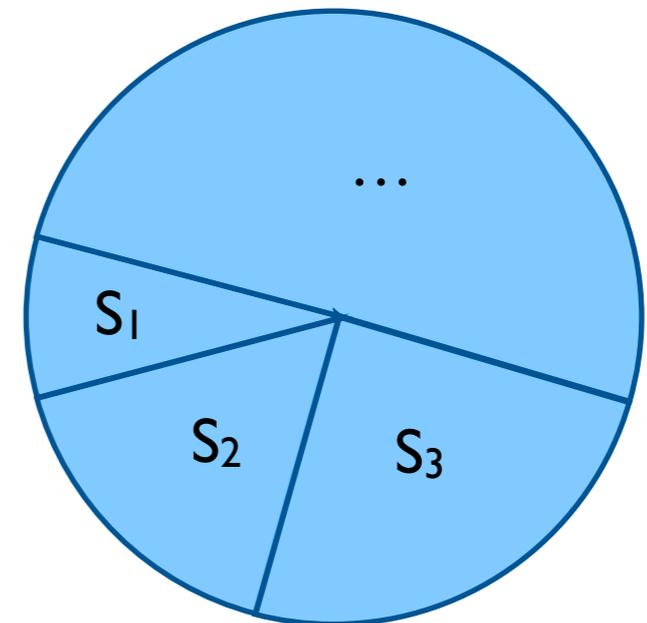
$\mathcal{S}, \preceq$  (SSA with iterative deepening)

## 2. cost function ( $\kappa$ )

▶  $\kappa : \{ P \mid P \in S_i \wedge S_i \in \mathcal{S} \} \rightarrow \mathbb{R}$

▶ assigns a numeric cost to each candidate

## 3. gradient function ( $g$ )



$$\kappa(P) = i \text{ for } P \in S_i \in \mathcal{S}$$

Counts the number of defined variables in the candidate program  $P$ .

# anatomy of a metasketch (2)

## 1. structured candidate space ( $\mathcal{S}, \preceq$ )

$\mathcal{S}, \preceq$  (SSA with iterative deepening)

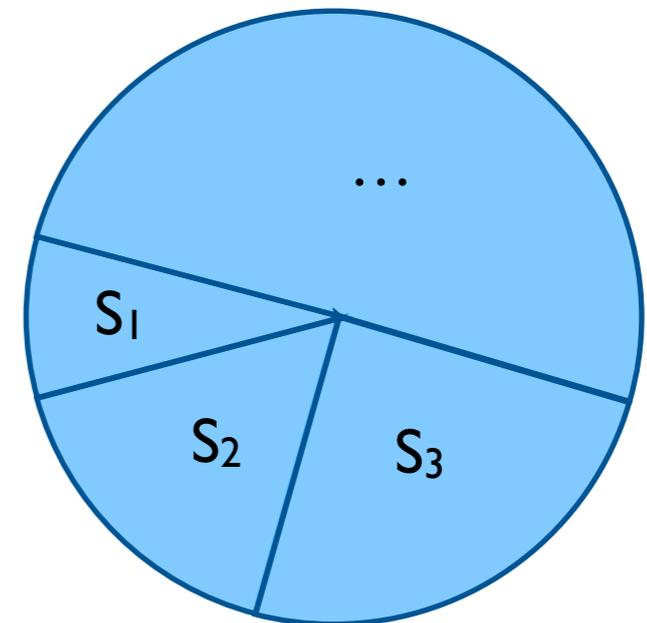
## 2. cost function ( $\kappa$ )

▶  $\kappa : \{ P \mid P \in S_i \wedge S_i \in \mathcal{S} \} \rightarrow \mathbb{R}$

▶ assigns a numeric cost to each candidate

## 3. gradient function ( $g$ )

The cost function  $\kappa$  can be *static* (based on program syntax) or *dynamic* (based on runtime behavior).



$$\kappa(P) = i \text{ for } P \in S_i \in \mathcal{S}$$

Counts the number of defined variables in the candidate program  $P$ .

# anatomy of a metasketch (2)

## 1. structured candidate space ( $\mathcal{S}, \preceq$ )

$\mathcal{S}, \preceq$  (SSA with iterative deepening)

## 2. cost function ( $\kappa$ )

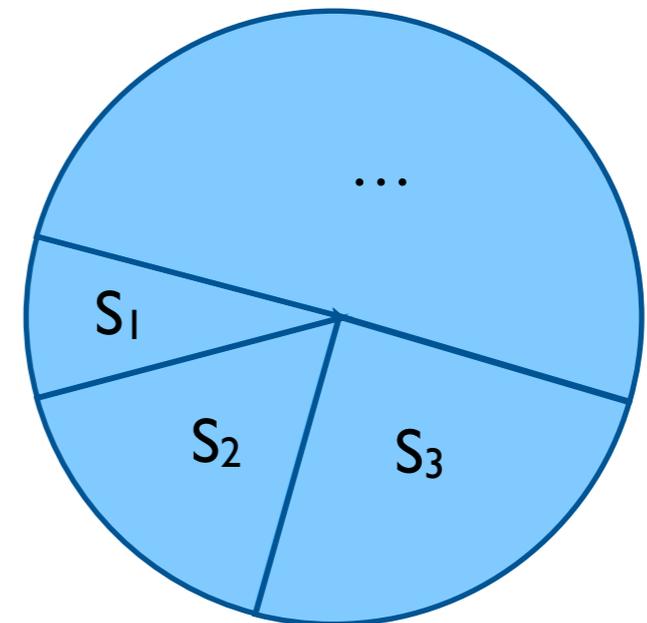
▶  $\kappa : \{ P \mid P \in S_i \wedge S_i \in \mathcal{S} \} \rightarrow \mathbb{R}$

▶ assigns a numeric cost to each candidate

## 3. gradient function ( $g$ )

The cost function  $\kappa$  can be *static* (based on program syntax) or *dynamic* (based on runtime behavior).

Any cost function  $\kappa$  can be used as long as the result of evaluating  $\kappa$  on a program  $P$  (and possibly its inputs) can be expressed as a term in a decidable theory.



$$\kappa(P) = i \text{ for } P \in S_i \in \mathcal{S}$$

Counts the number of defined variables in the candidate program  $P$ .

# anatomy of a metasketch (3)

1. structured candidate space ( $\mathcal{S}, \preceq$ )

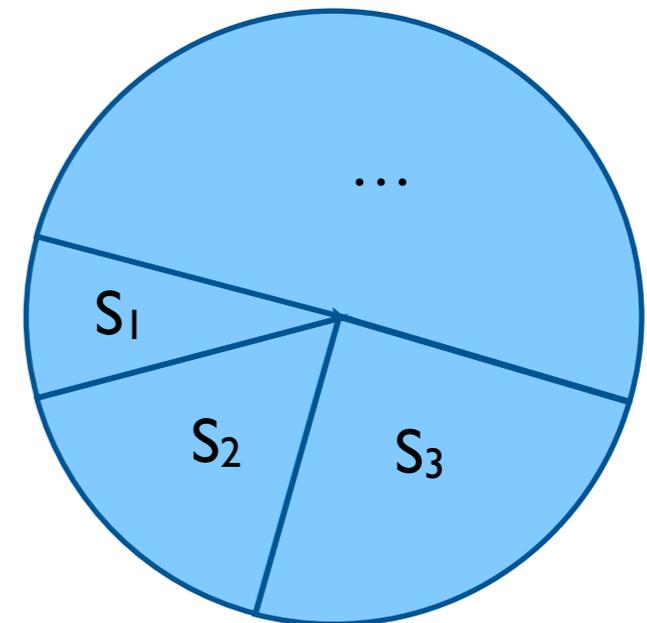
$\mathcal{S}, \preceq$  (SSA with iterative deepening)

2. cost function ( $\kappa$ )

3. gradient function ( $g$ )

▶  $g: \mathbb{R} \rightarrow 2^{\mathcal{S}}$

▶  $g(c)$  is the set of all sketches in  $\mathcal{S}$  that may contain a program  $P$  with  $\kappa(P) < c$



$\kappa(P) = i$  for  $P \in S_i \in \mathcal{S}$

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2. cost function ( $\kappa$ )

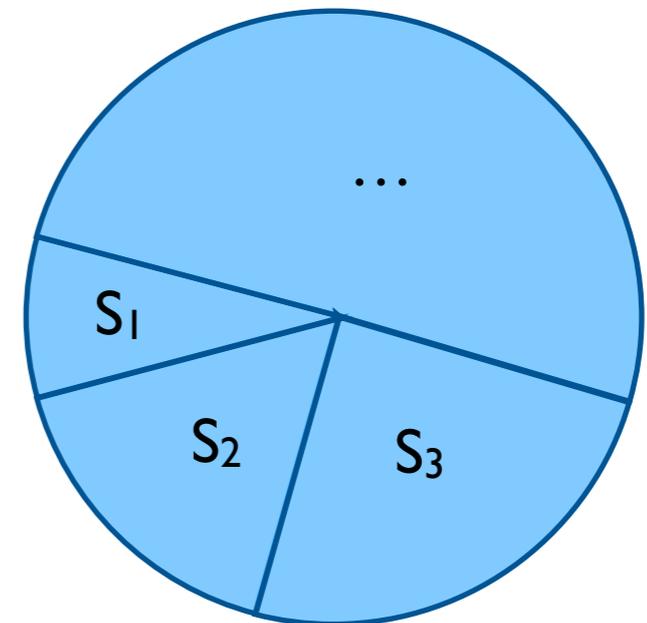
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The gradient function  $g$  overapproximates the behavior of  $\kappa$  on  $\mathcal{S}$ .

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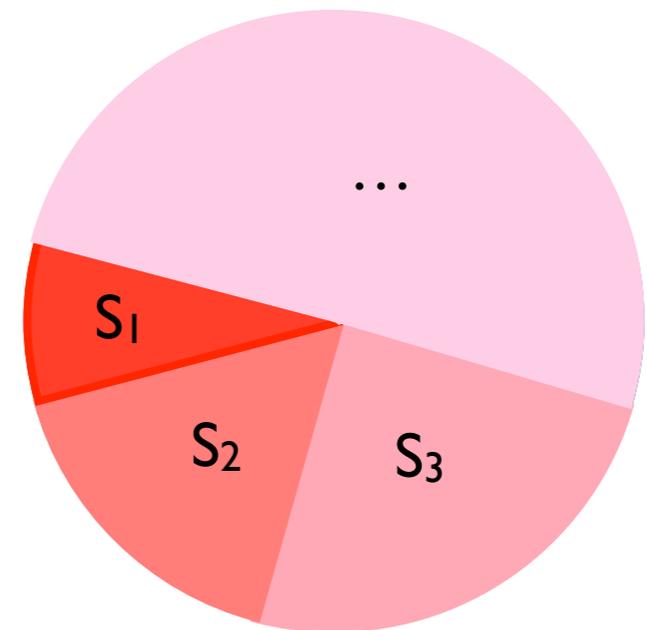
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$g(c) = \{ S_i \in \mathcal{S} \mid i < c \}$

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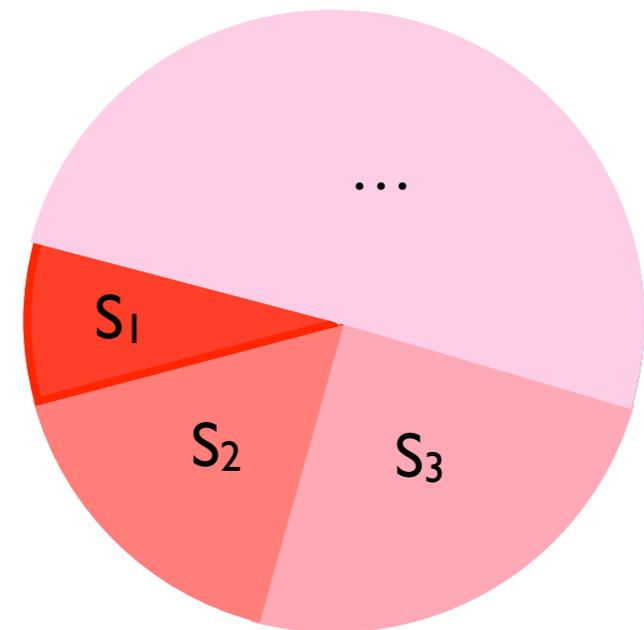
▶  $g: \mathbb{R} \rightarrow 2^{\mathcal{S}}$

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The gradient function  $g$  overapproximates the behavior of  $\kappa$  on  $\mathcal{S}$ .

It is always sound to return the trivial gradient  $g(c) = \mathcal{S}$ .

$\mathcal{S}, \preceq$  (SSA with iterative deepening)

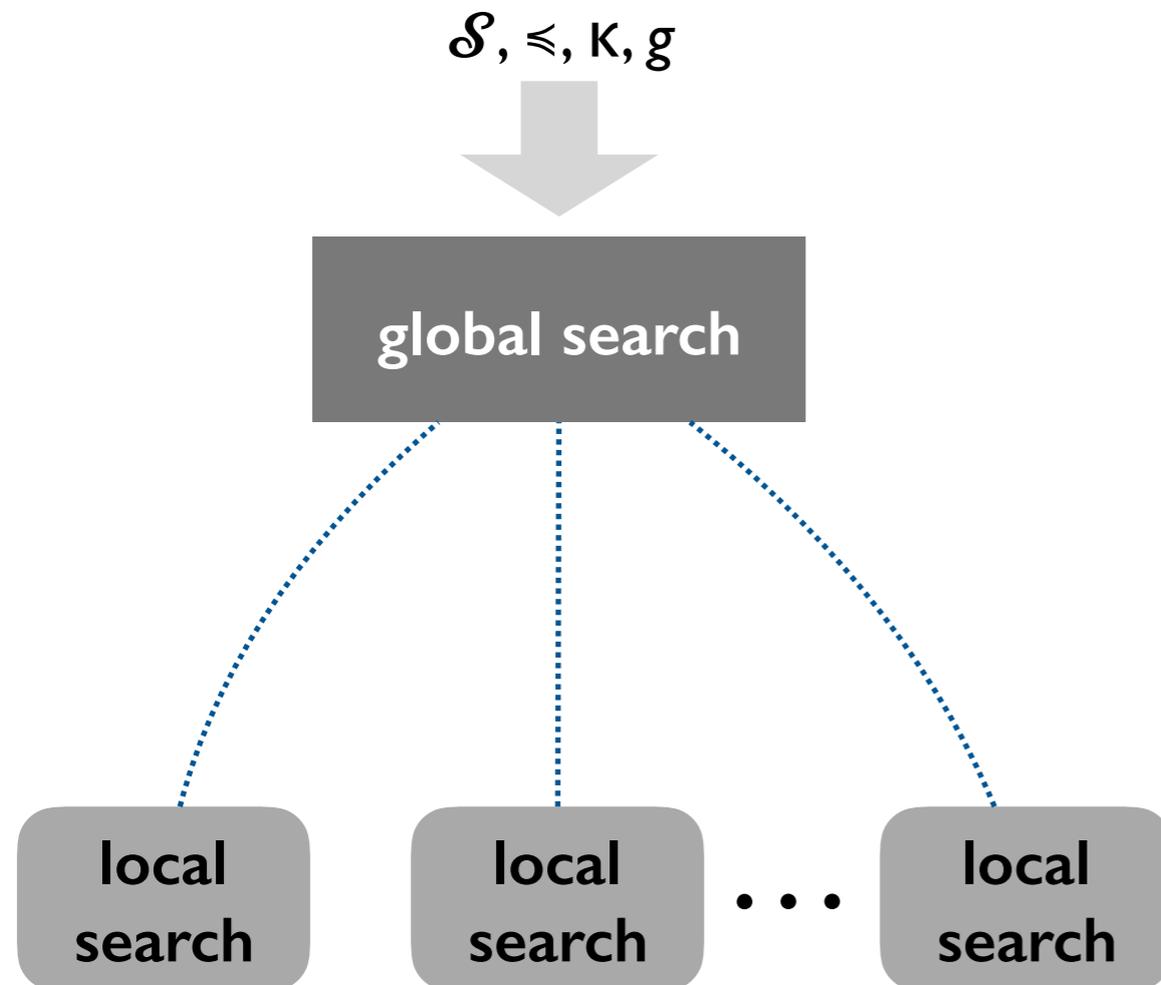


$\kappa(P) = i$  for  $P \in S_i \in \mathcal{S}$

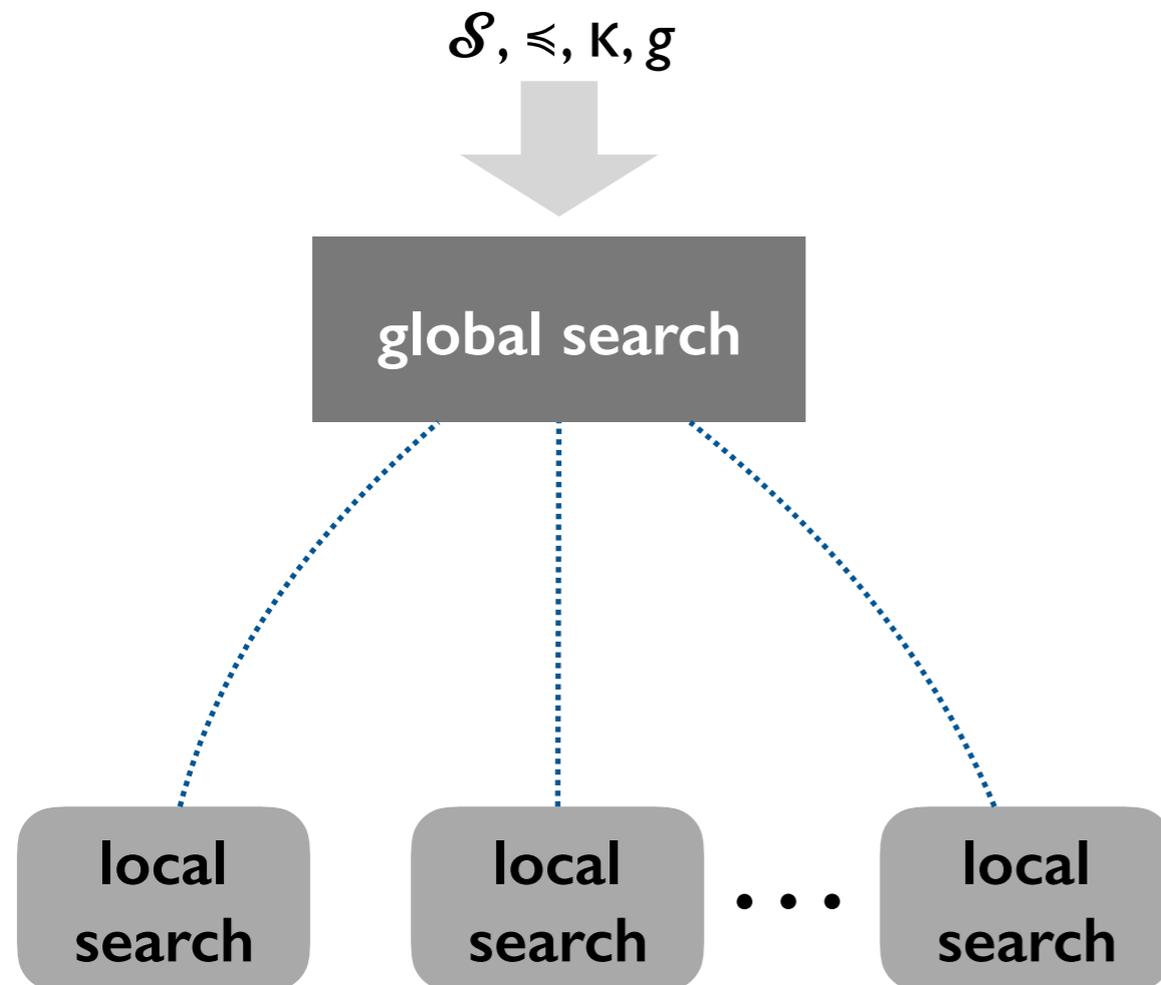
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# basic idea: two cooperating search algorithms

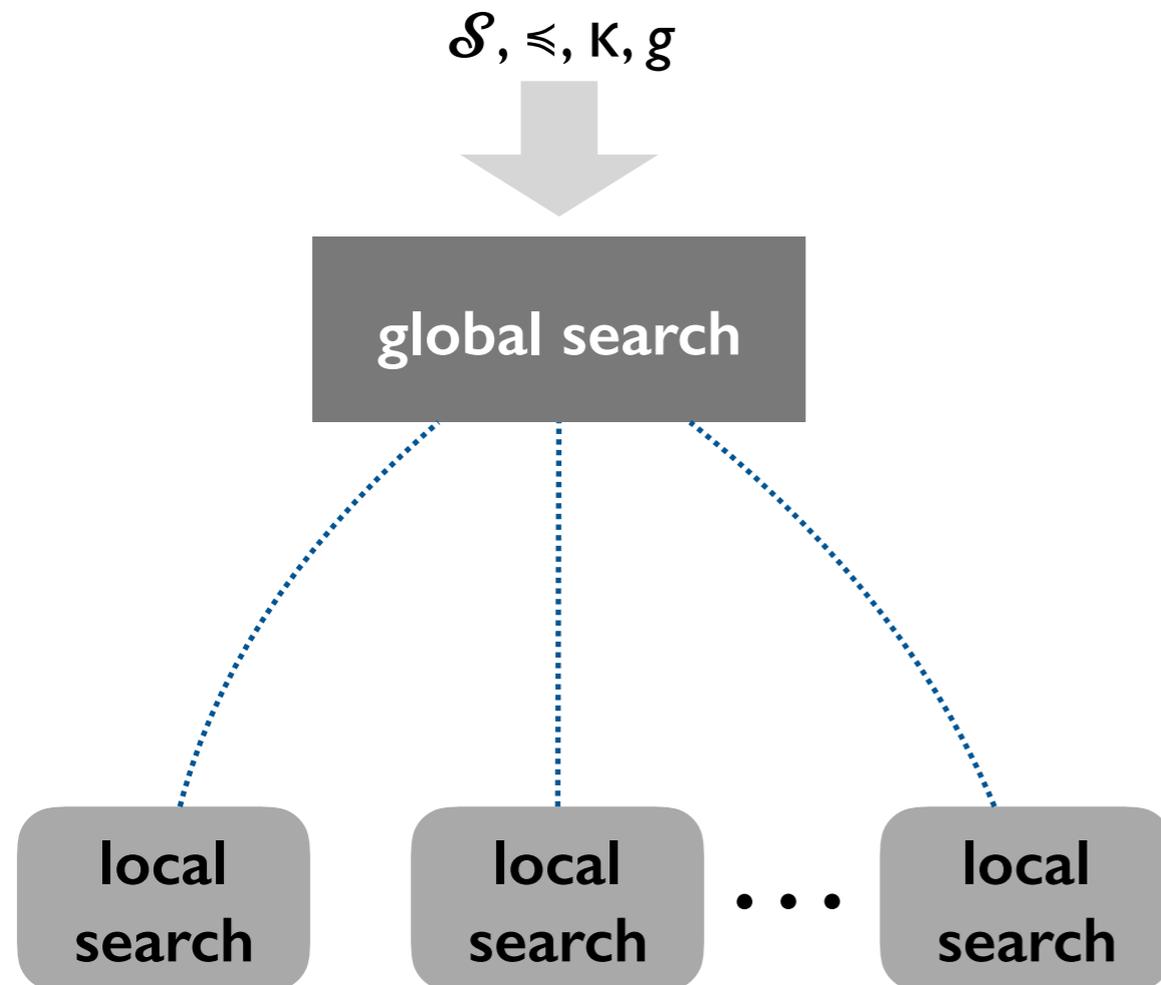


# basic idea: two cooperating search algorithms



*Global optimizing search*  
coordinates the activities of local  
searches running in parallel on  
individual sketches in  $\mathcal{S}$ .

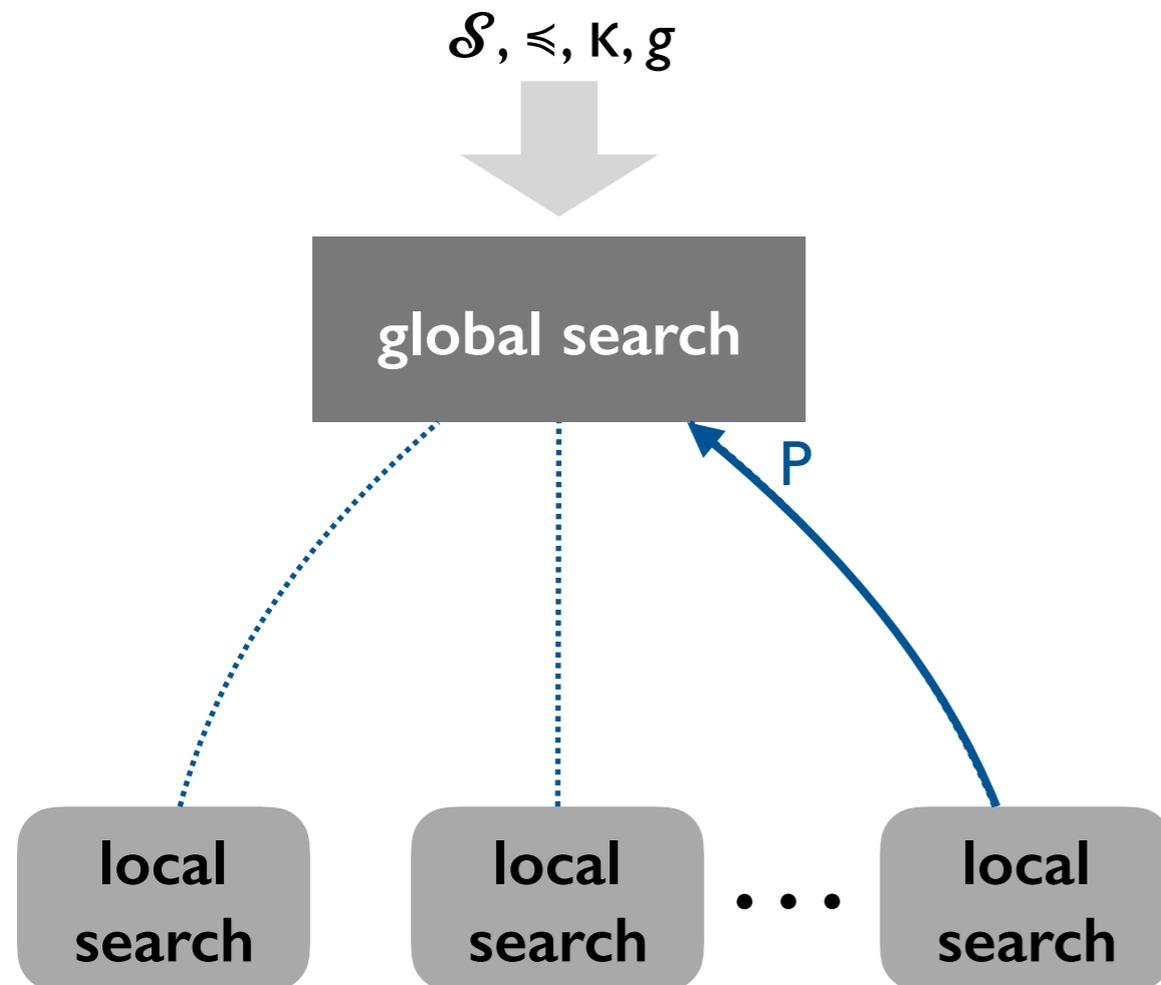
# basic idea: two cooperating search algorithms



*Global optimizing search* coordinates the activities of local searches running in parallel on individual sketches in  $\mathcal{S}$ .

*Local combinatorial search* employs an incremental form of CEGIS to incorporate the information sent by the global search.

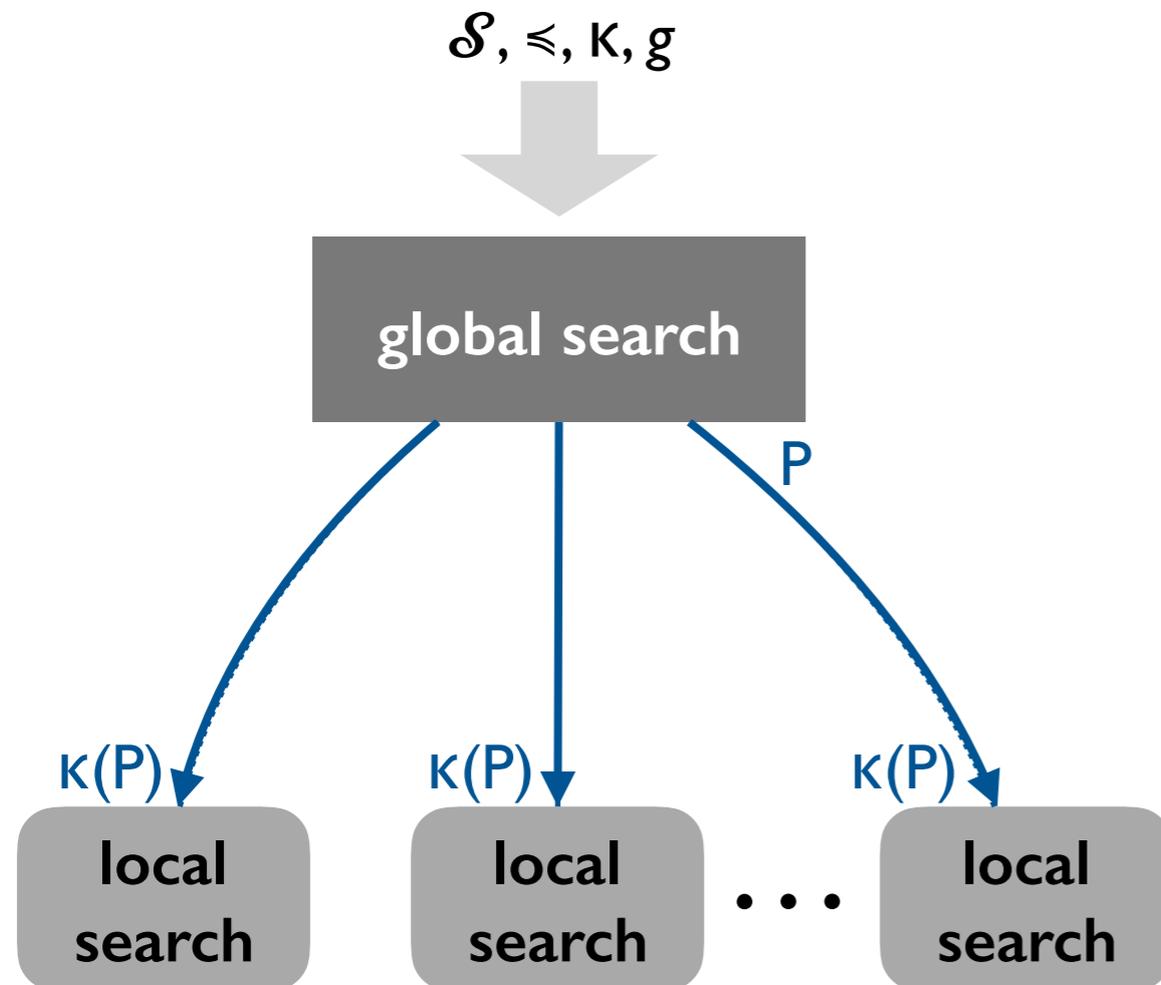
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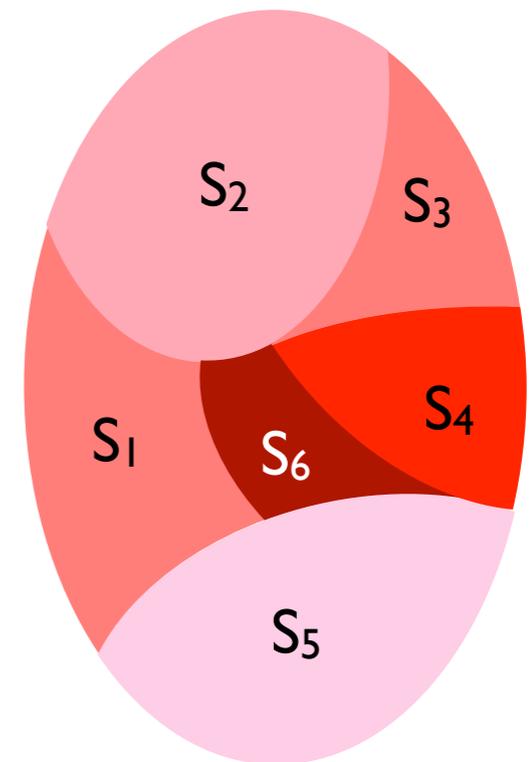
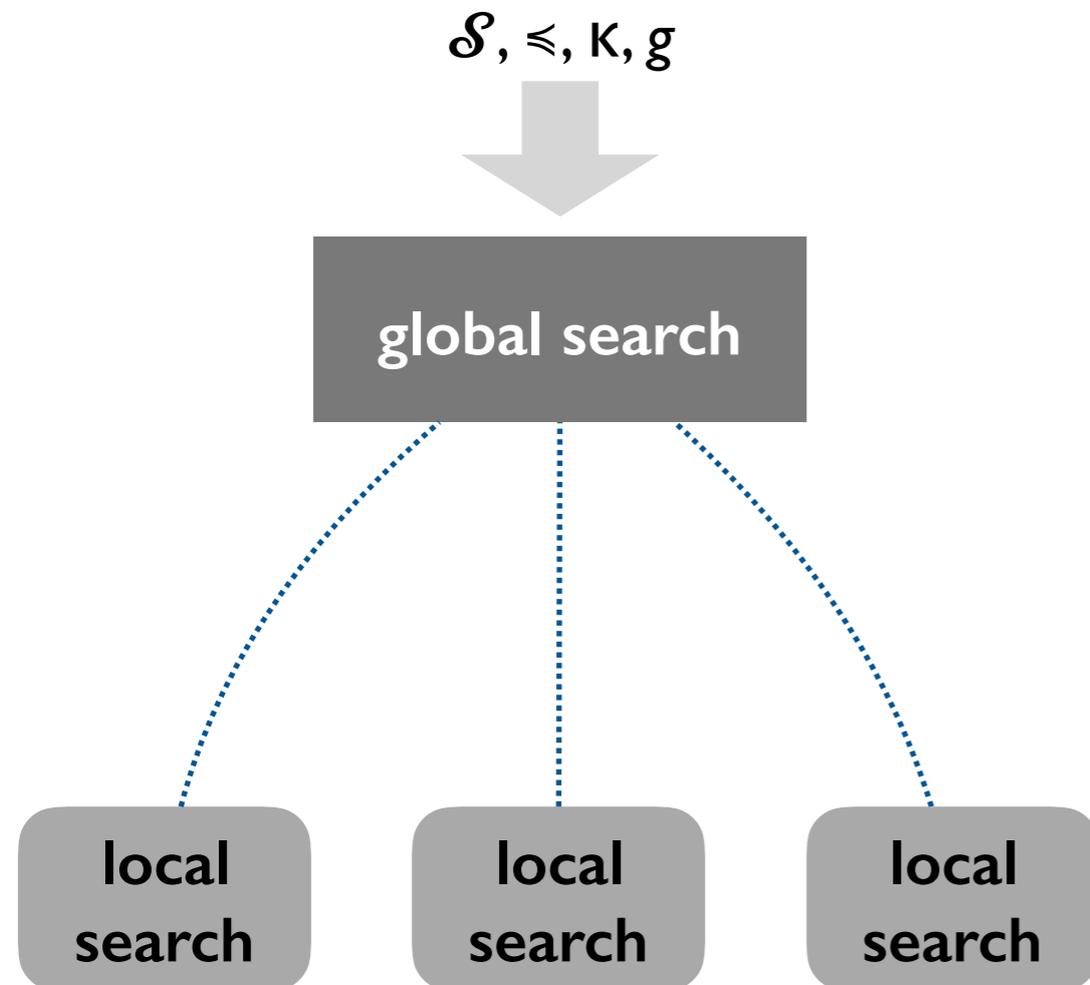
# basic idea: two cooperating search algorithms



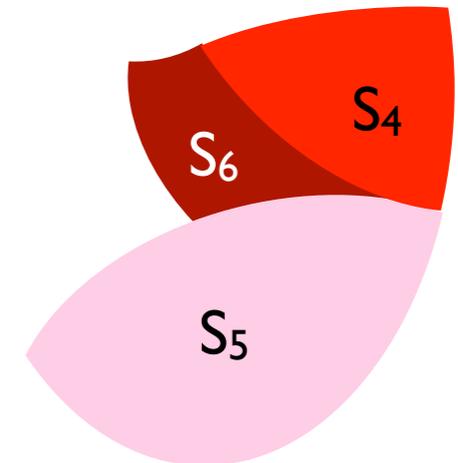
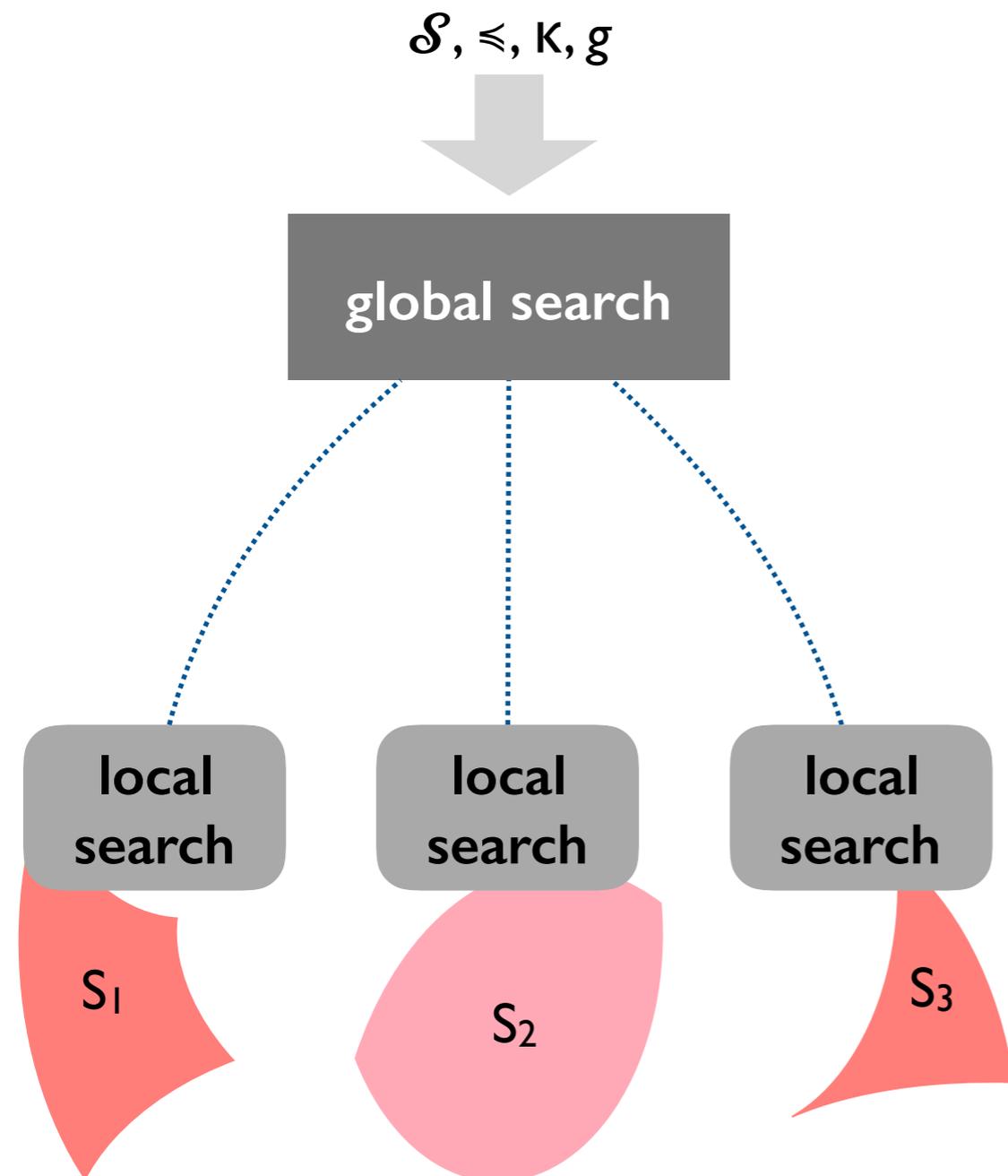
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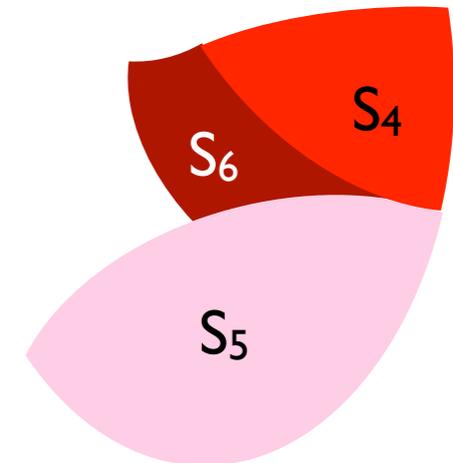
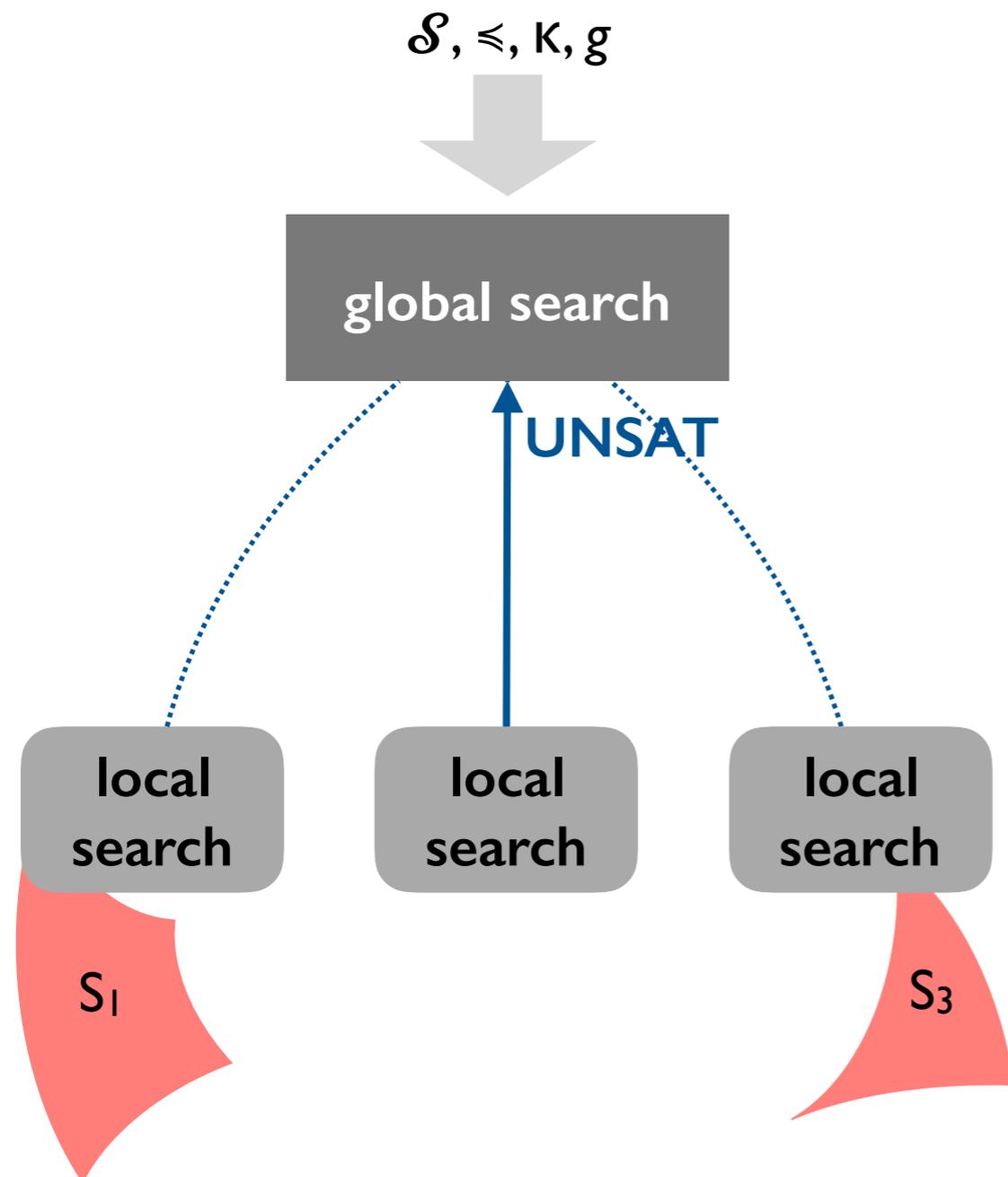
# synapse by example



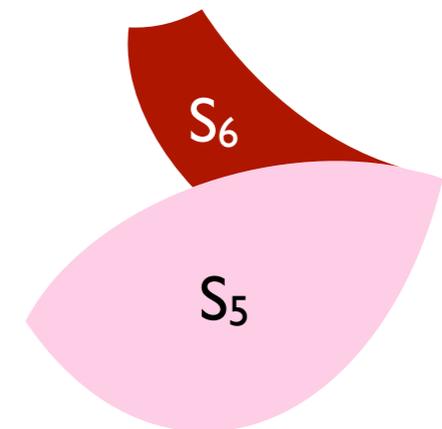
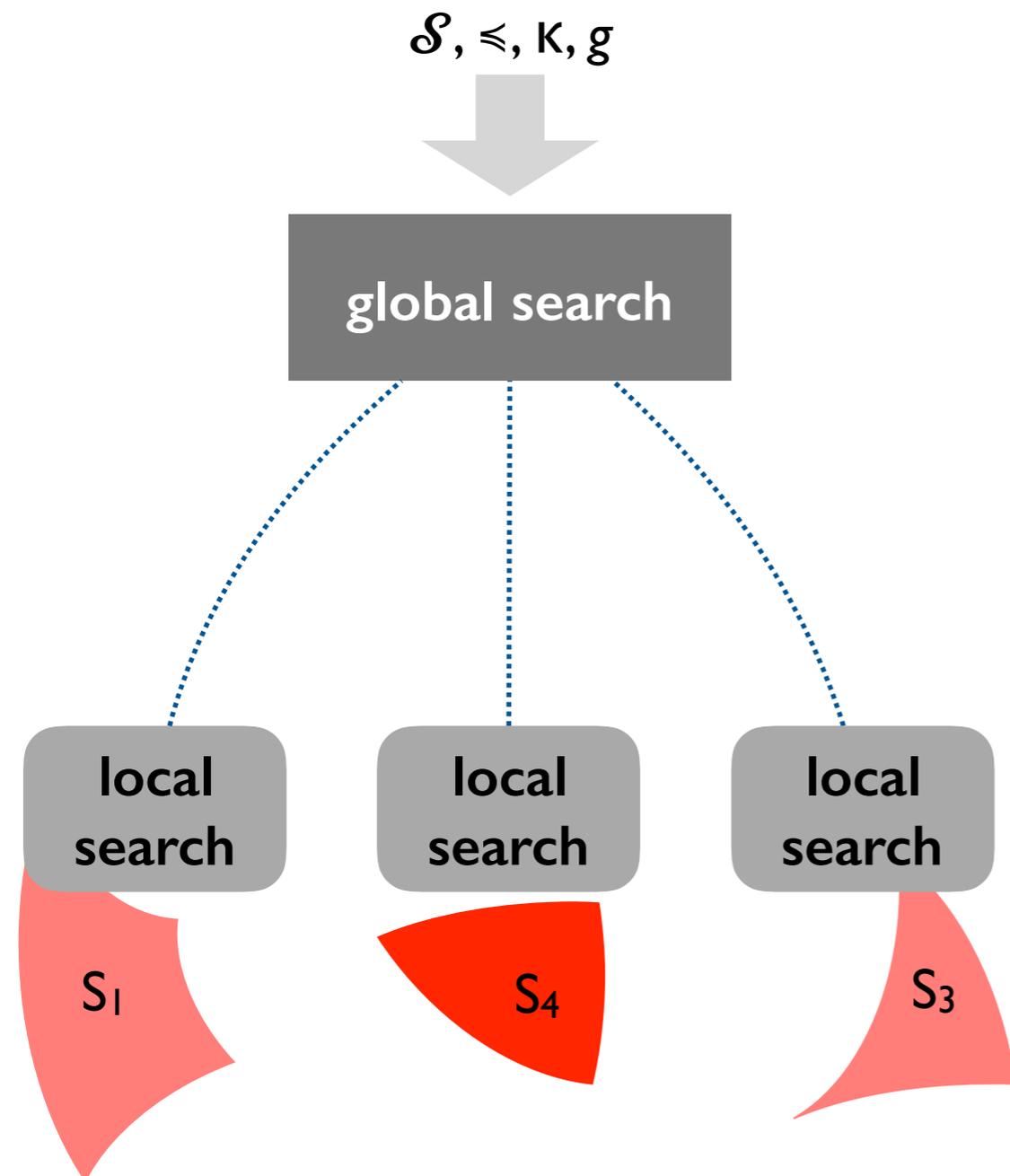
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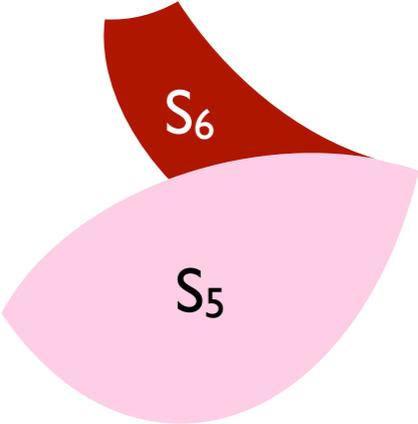
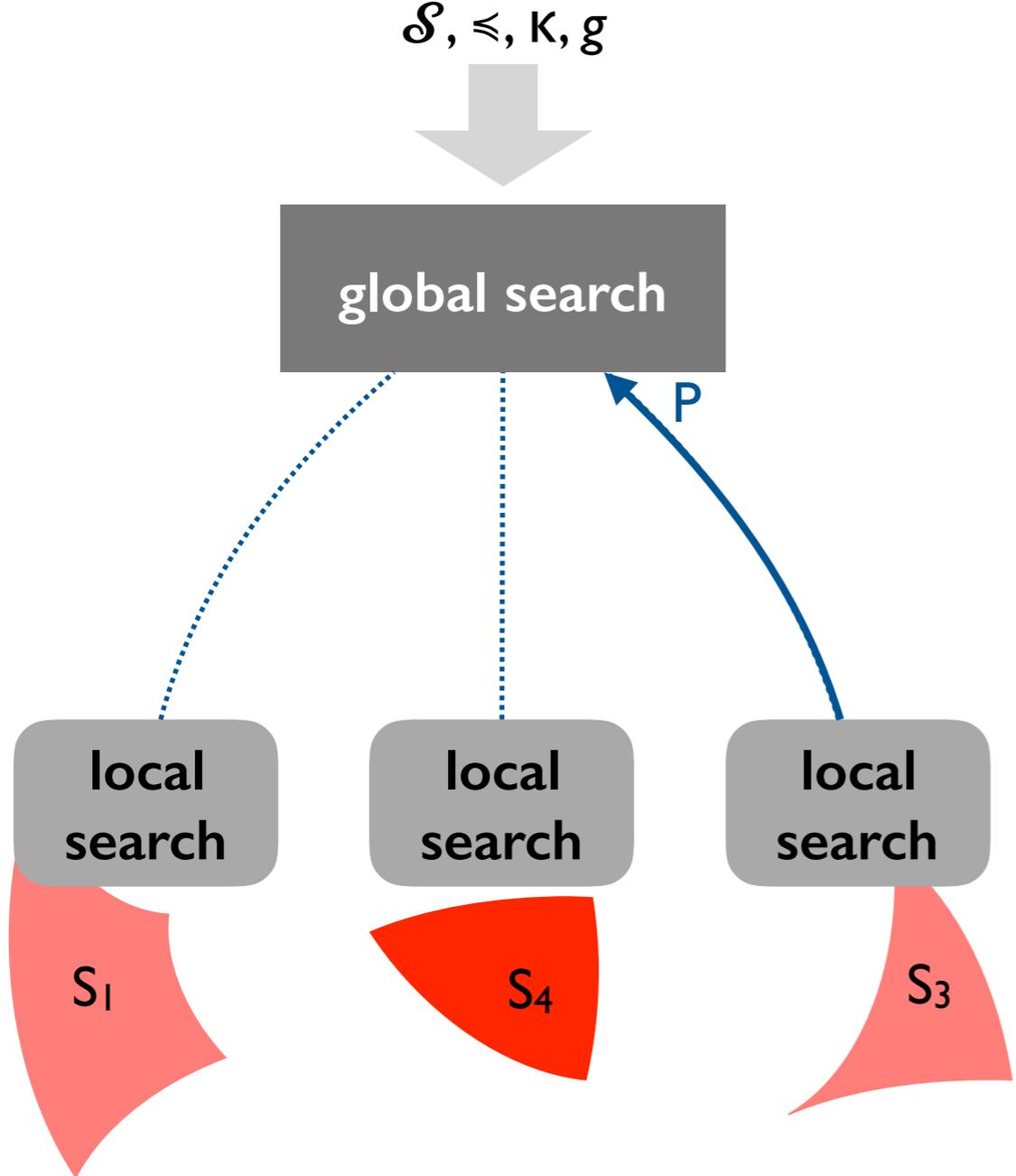
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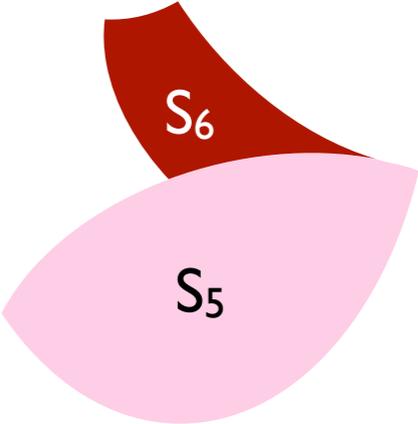
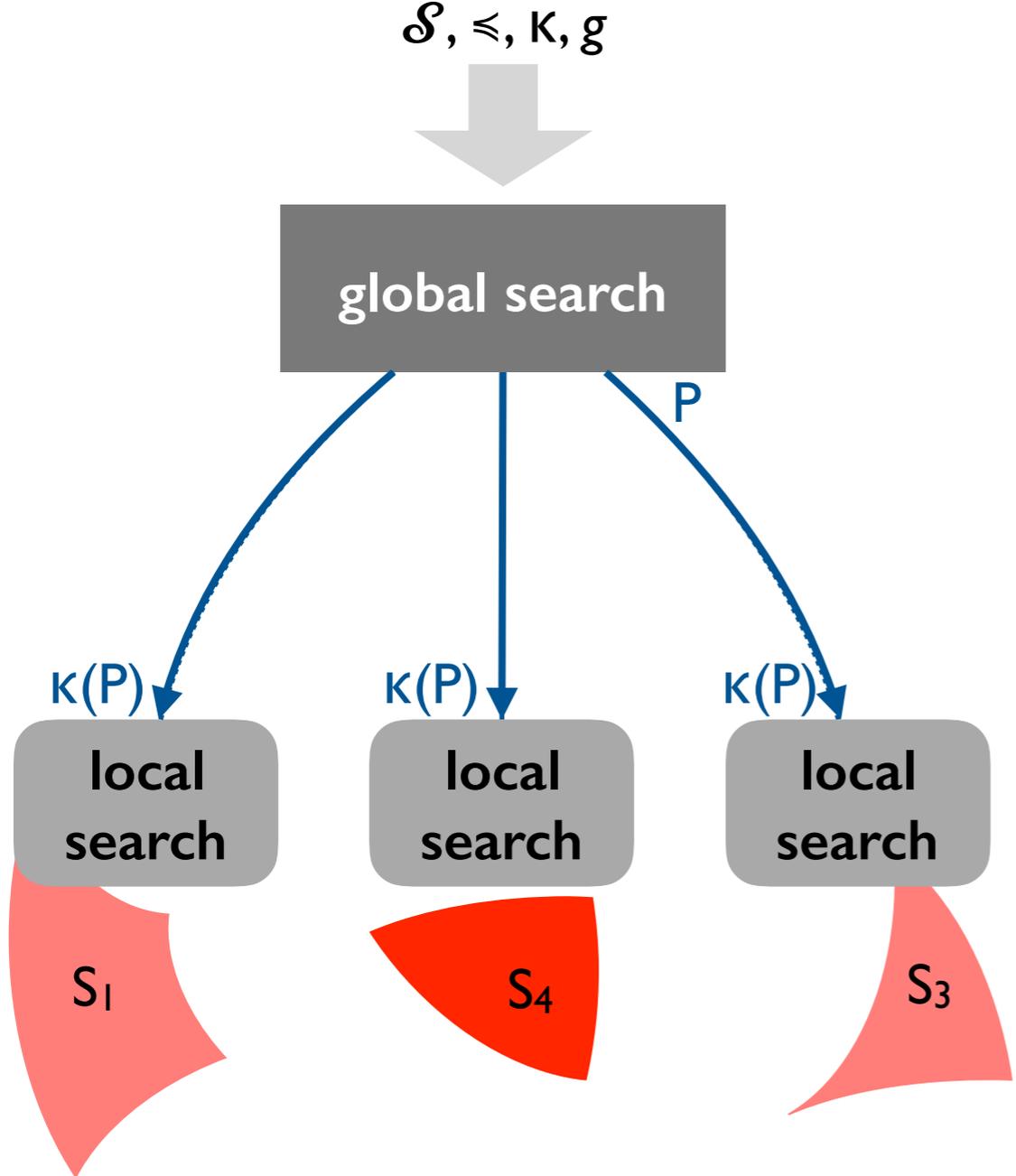
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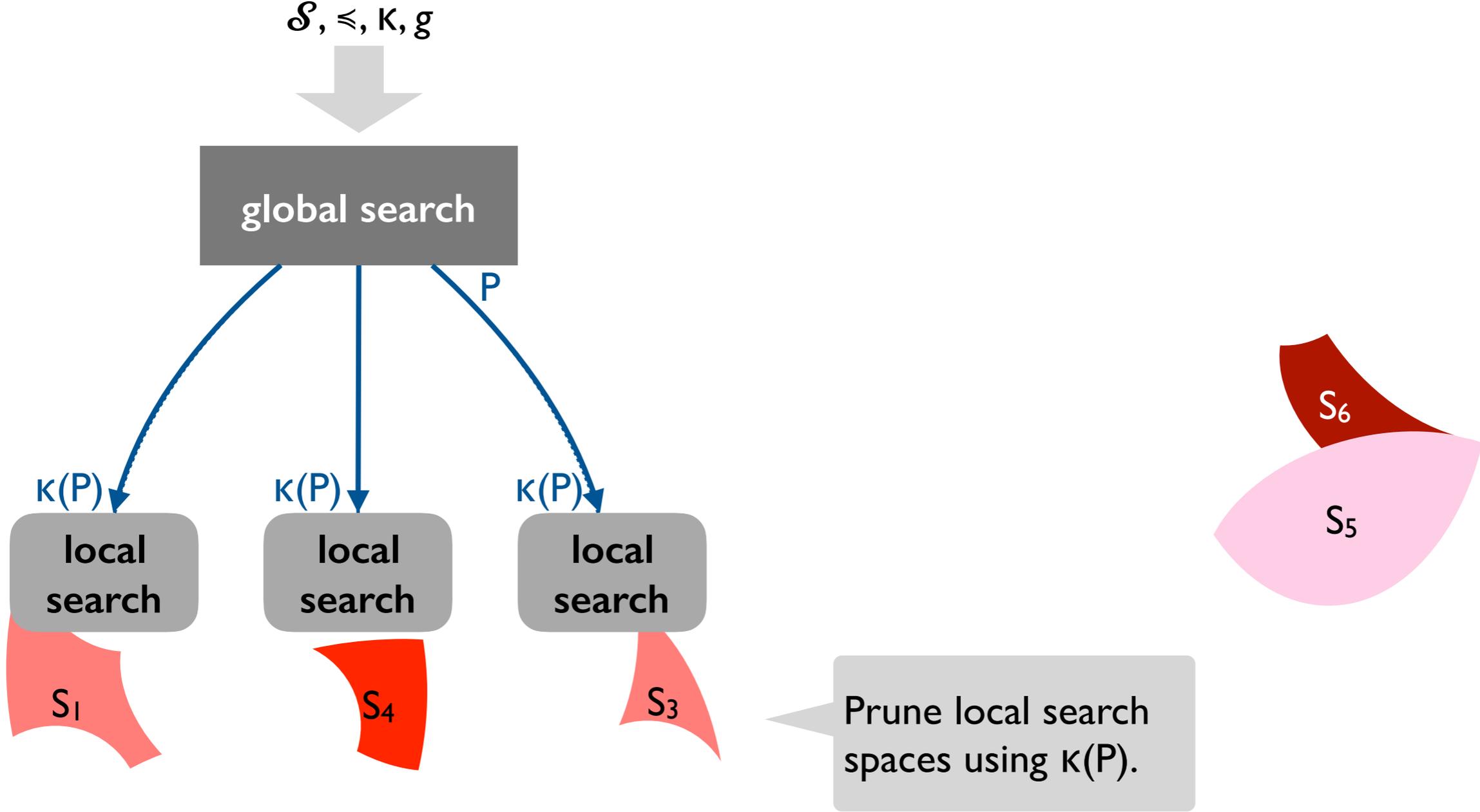
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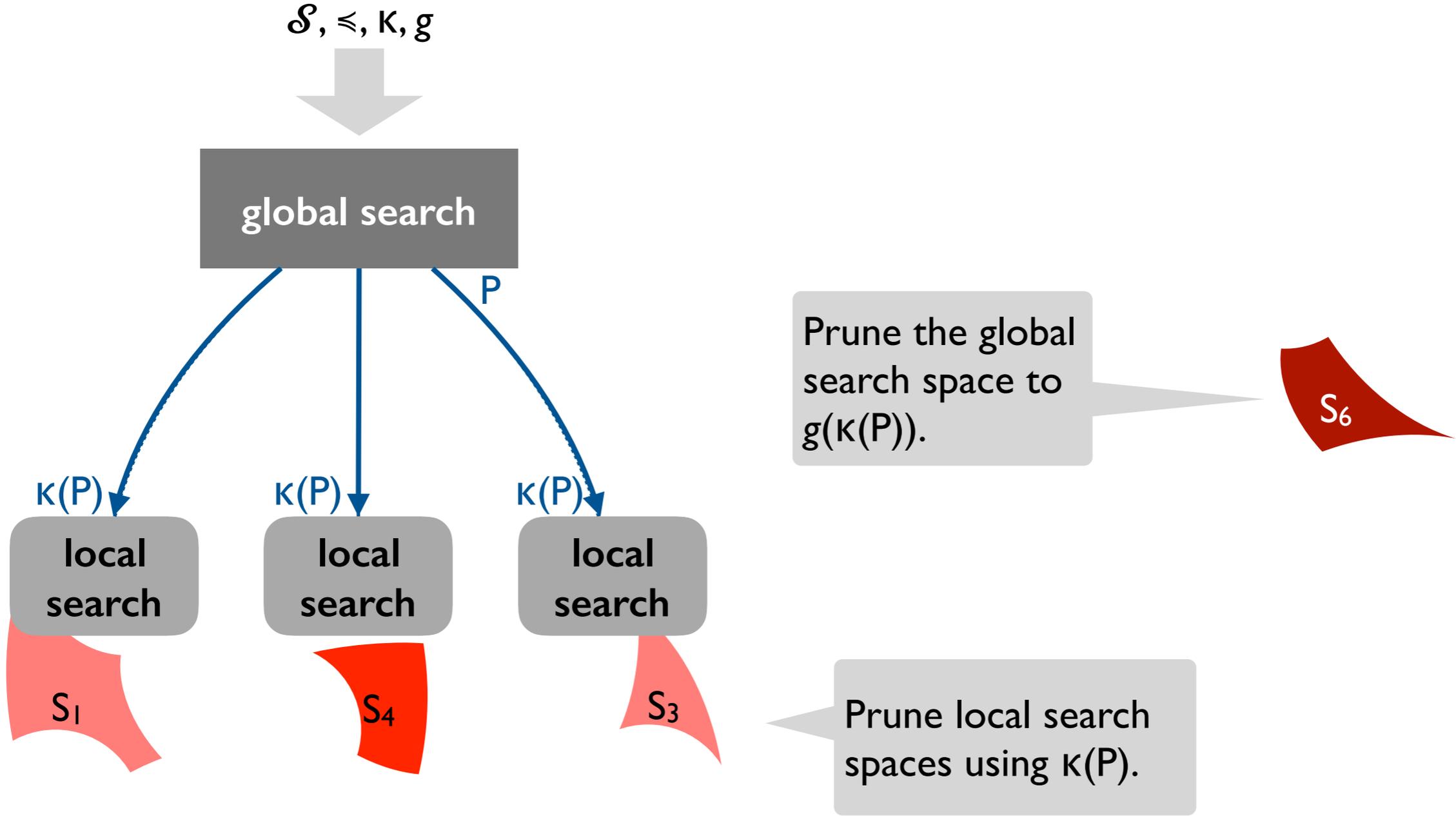
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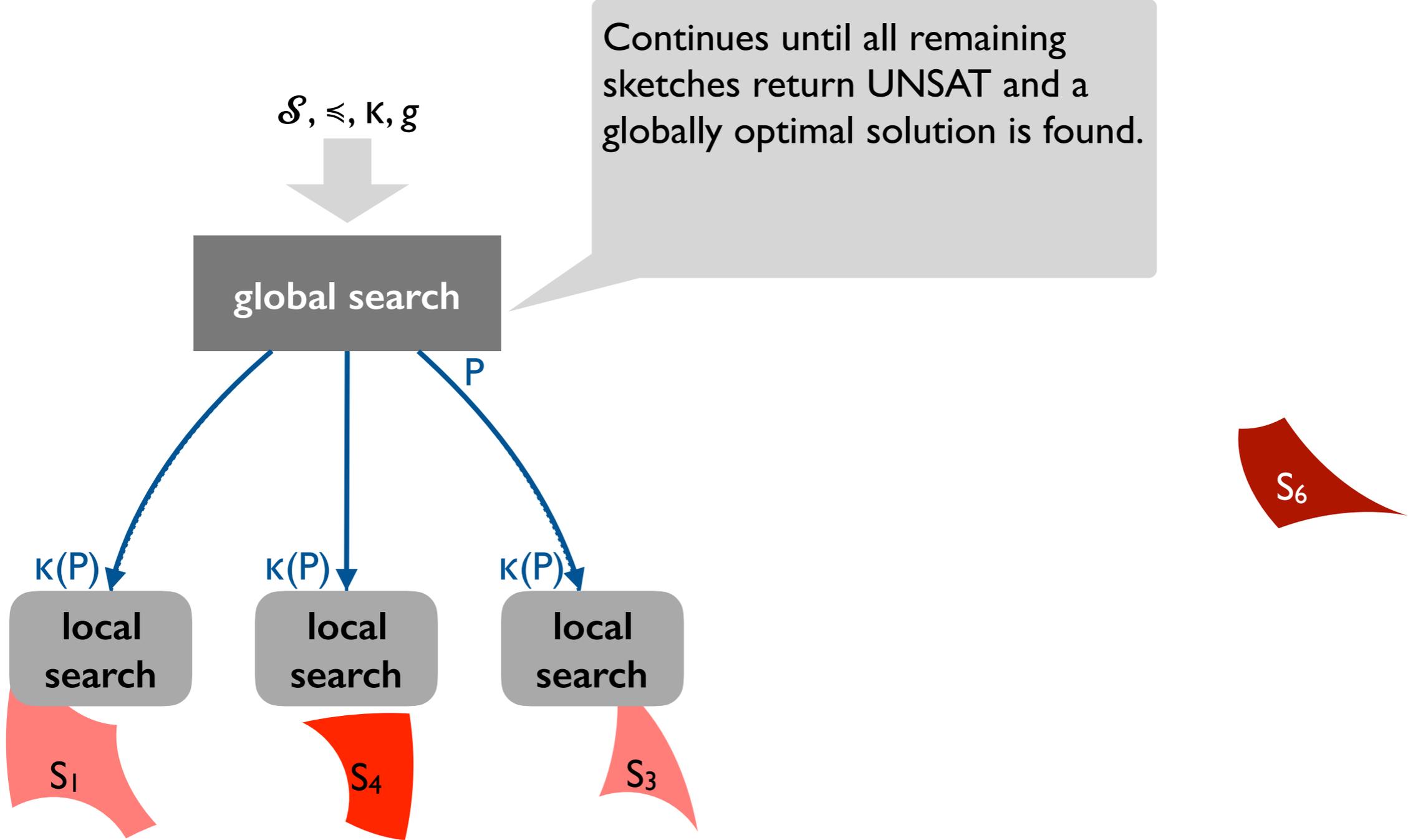
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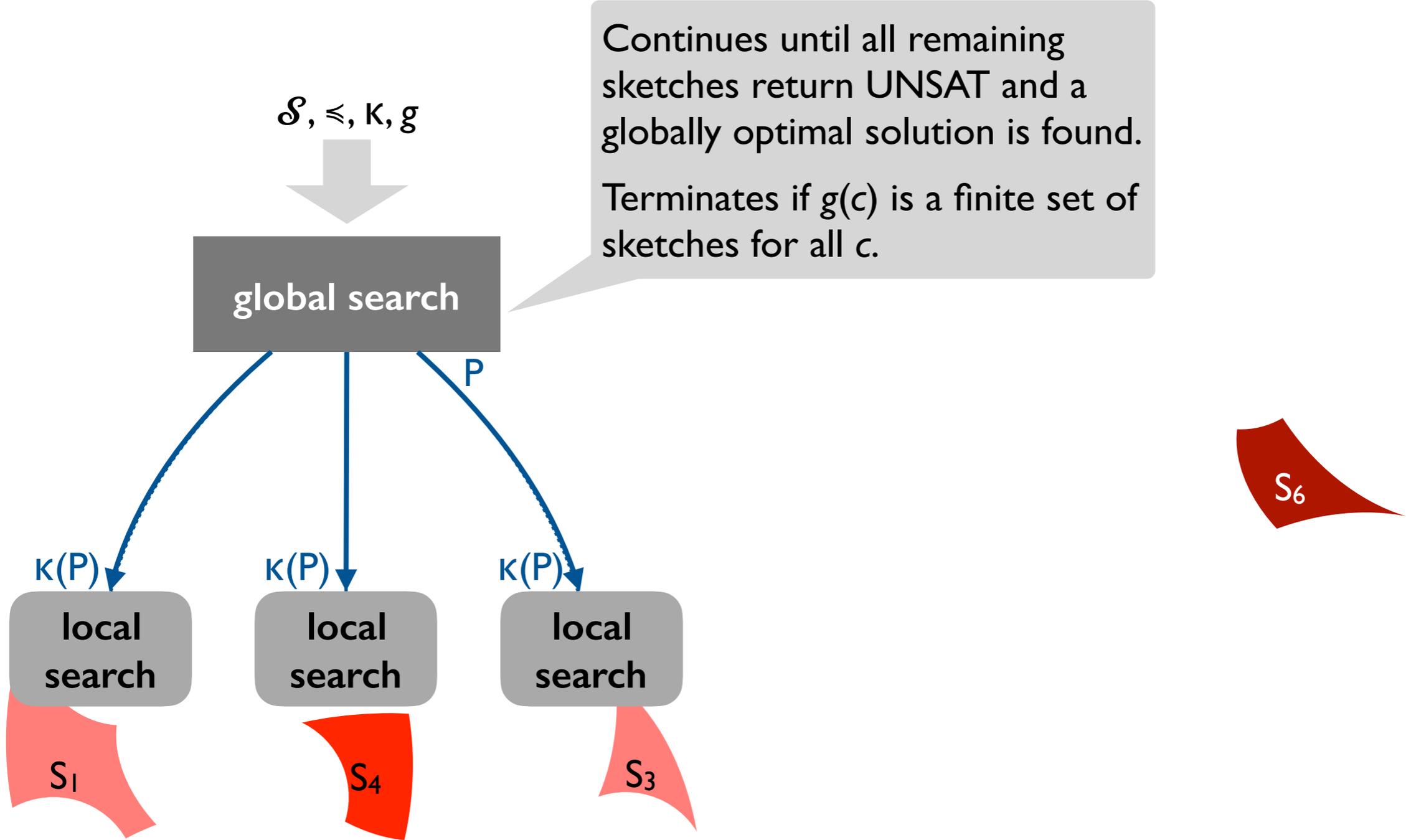
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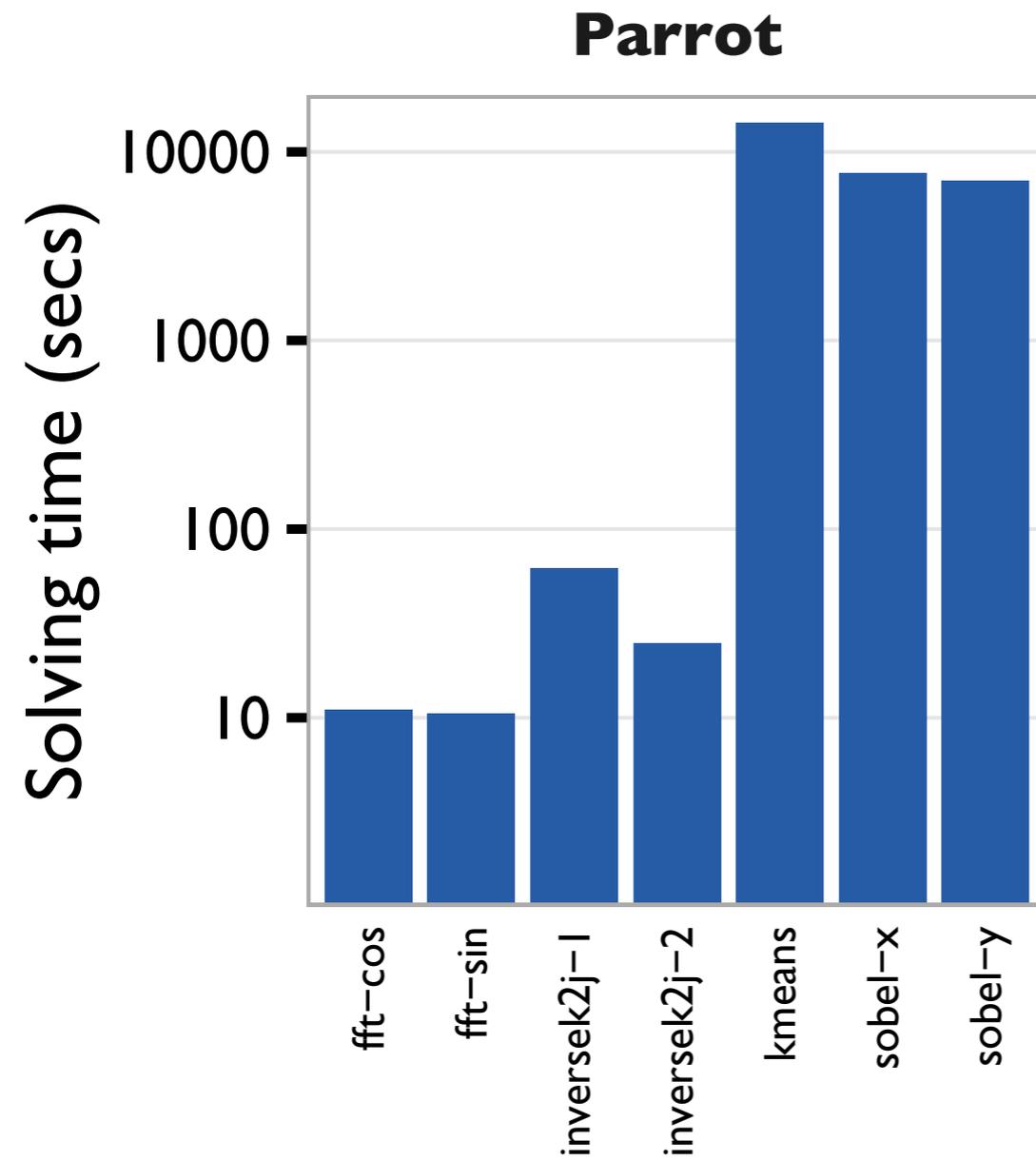
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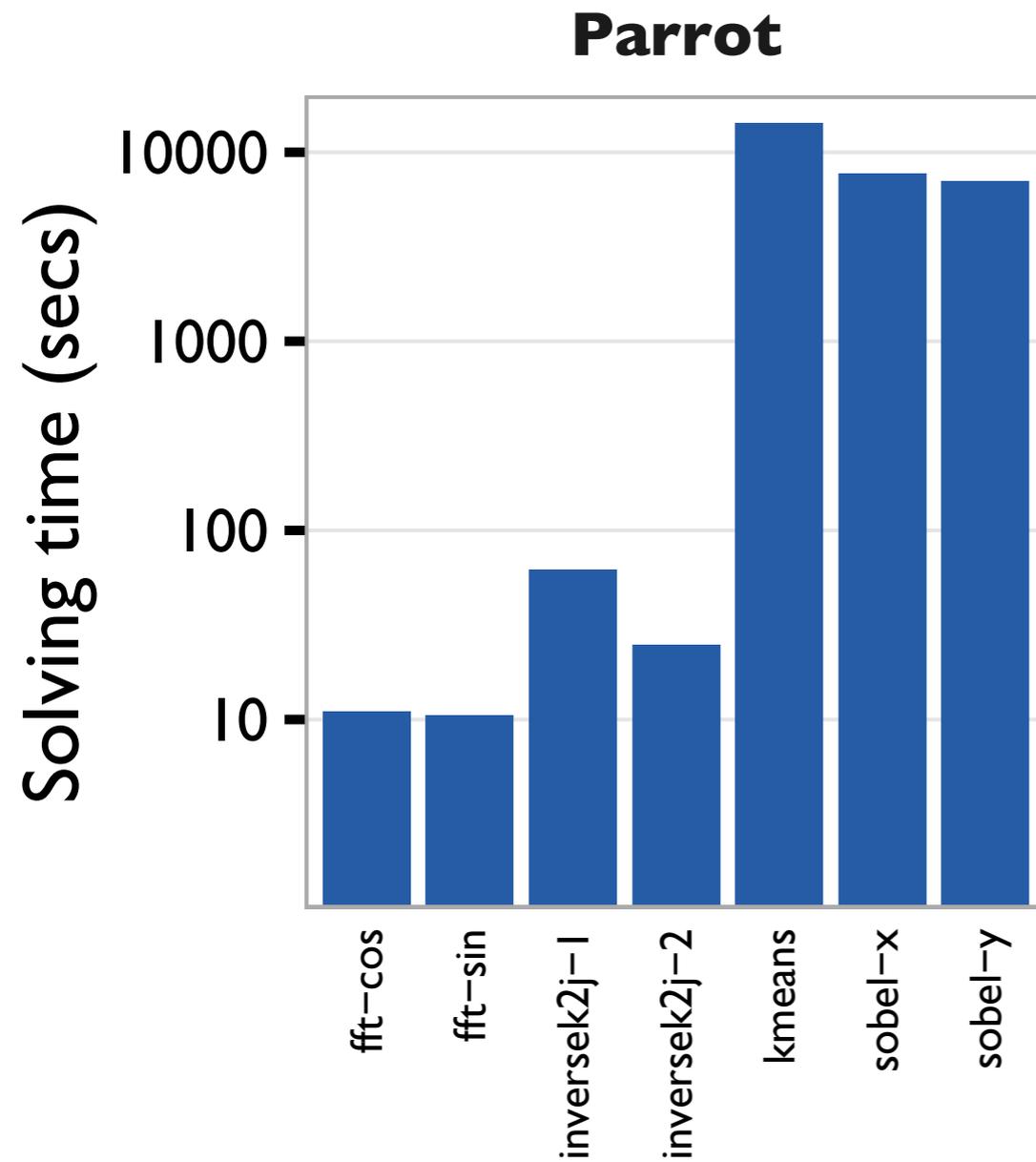
results

**evaluation**

# synapse can solve new classes of problems

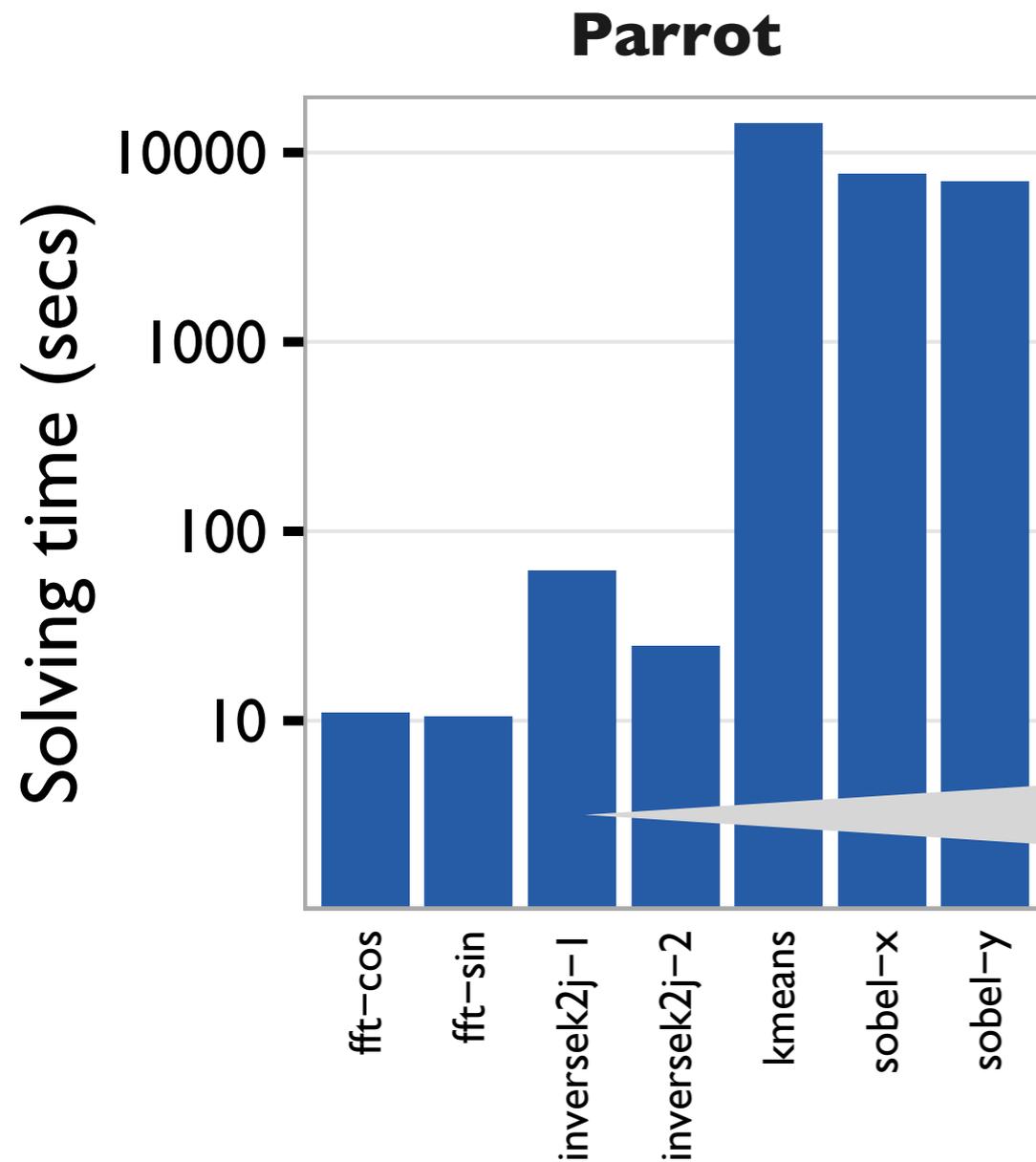


# synapse can solve new classes of problems



Finds the cheapest program that approximates a given reference program with respect to an application-specific error bound.

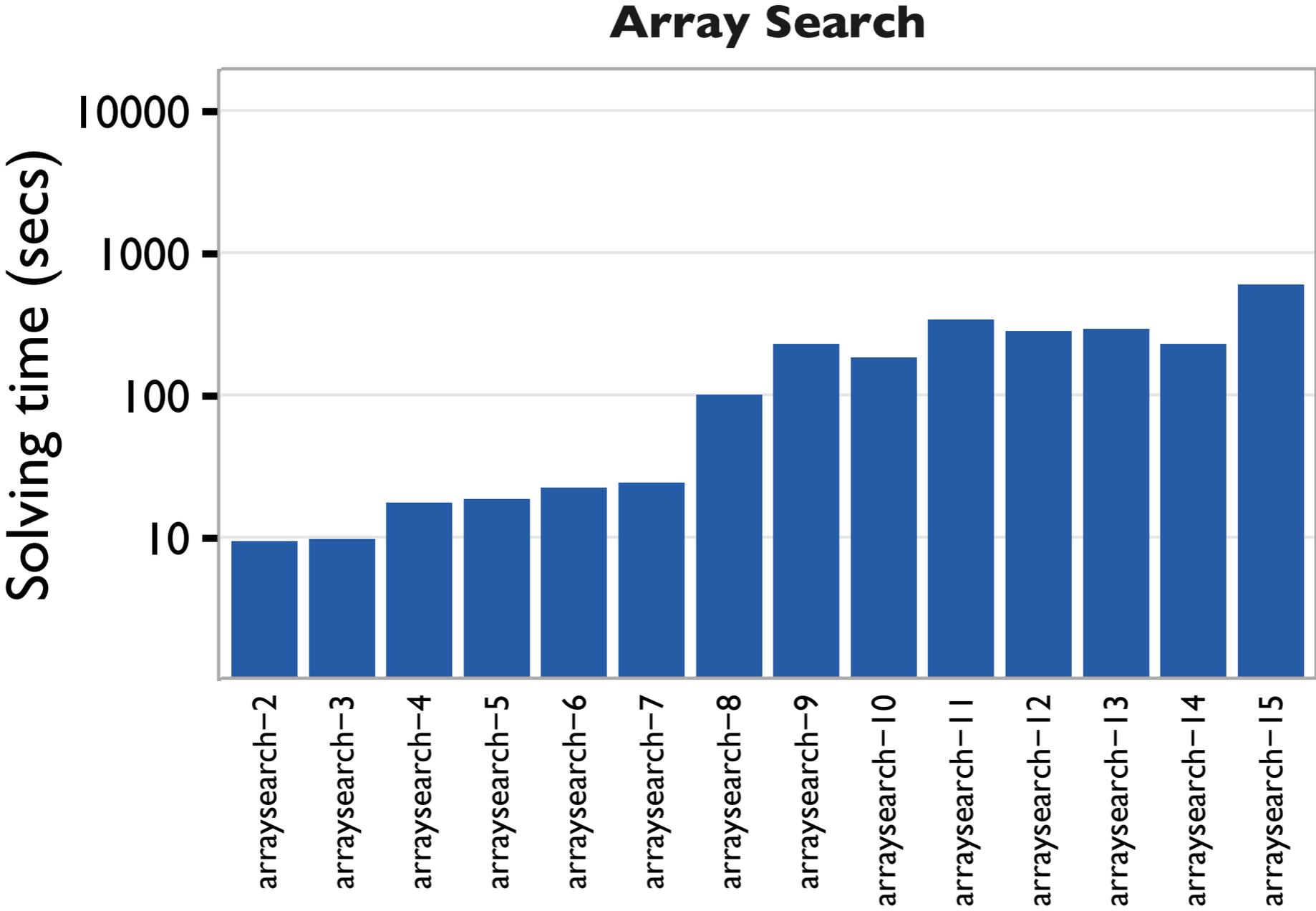
# synapse can solve new classes of problems



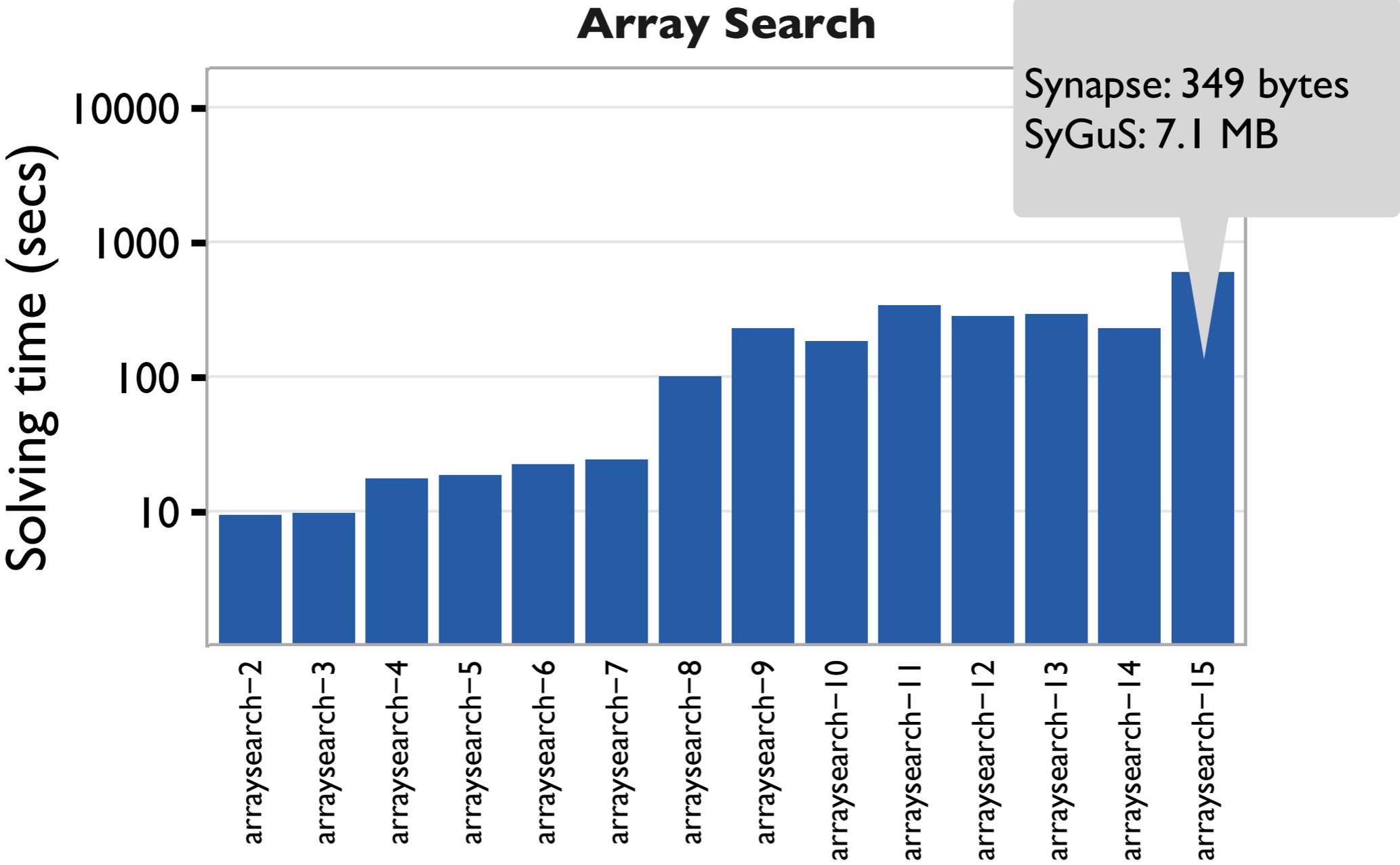
Finds the cheapest program that approximates a given reference program with respect to an application-specific error bound.

```
def inversek2j(float x, float y):  
    th2 = acos(((x*x) + (y*y) - 0.5) / 0.5)  
    th1 = asin((y * (0.5 + 0.5*cos(*th2)) -  
                0.5*x*sin(*th2)) /  
                (x*x + y*y))  
  
    return th1
```

# synapse solves standard benchmarks optimally



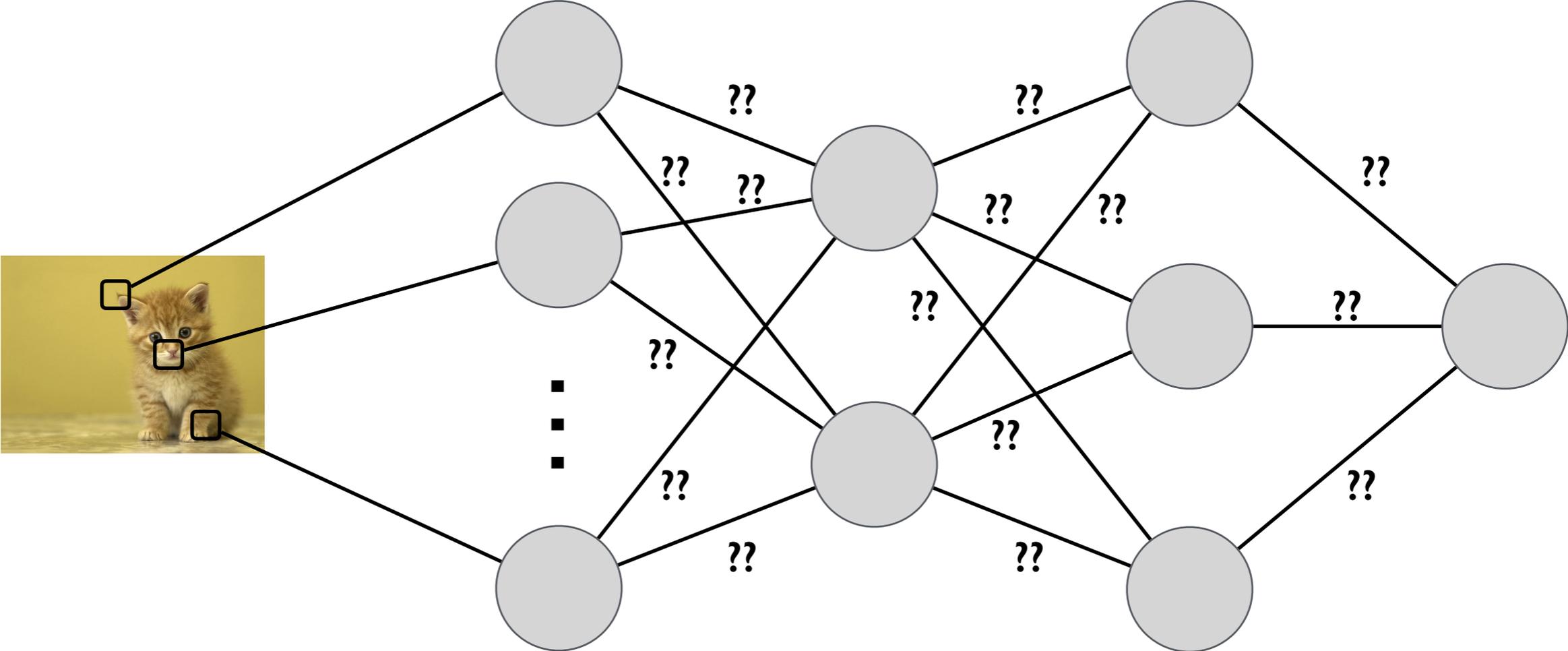
# synapse solves standard benchmarks optimally



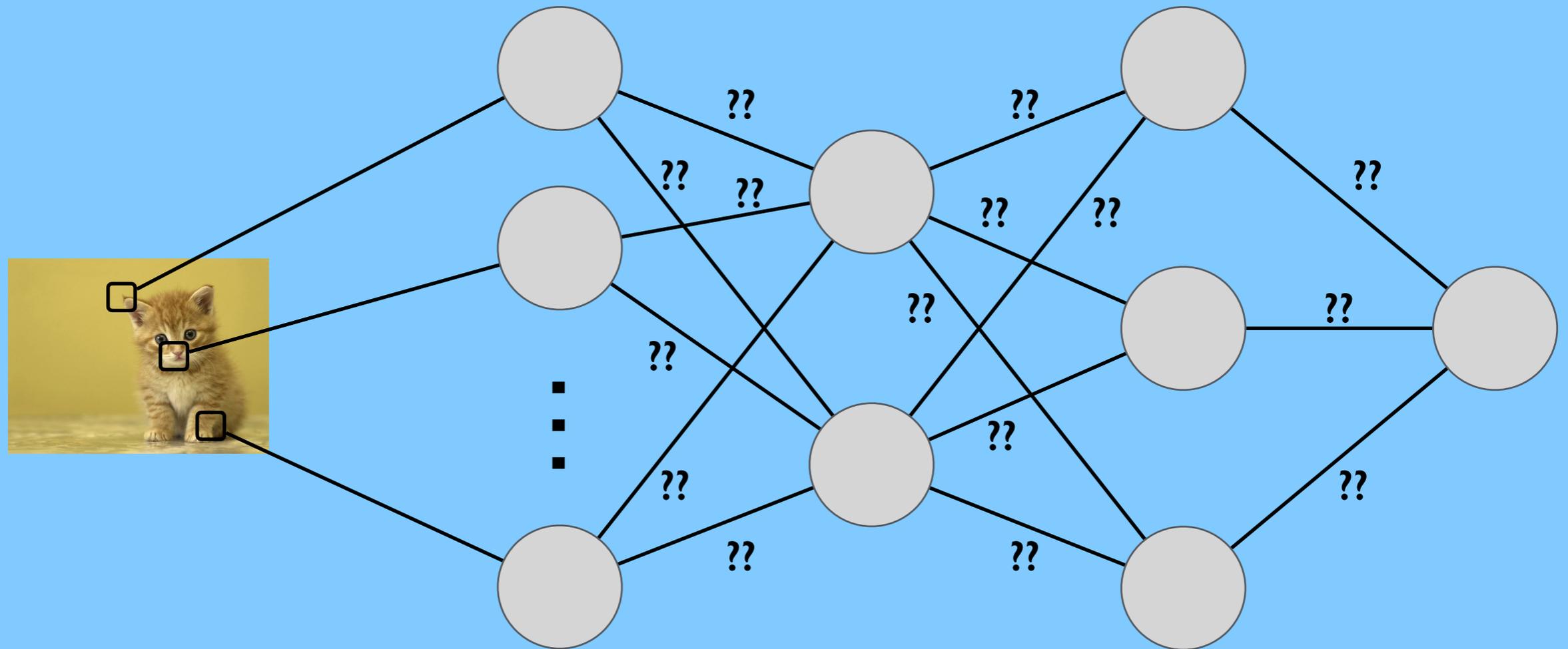
**is this a cat?**



# synapse can reason about complex costs



# synapse can reason about complex costs



$\mathcal{S}, \preceq$  : a finite set of network topologies

$$\kappa(P) = \sum_i |P(x_i) - y_i|$$

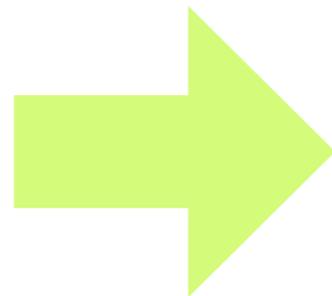
$$g(c) = \mathcal{S}$$

$\mathcal{S}, \approx, K, g$

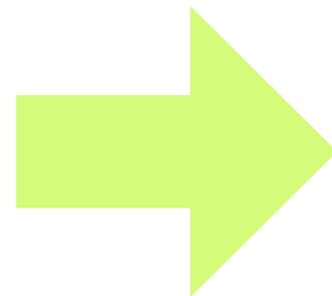


thanks!

<http://synapse.uwplse.org>



is this a cat?



yes!